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Mathematical Problem Posing and Metacognition: A Theoretical Framework

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Abstract: Mathematics education is moving toward student-centred which is beyond problem solving abilities and can become skilful in problem posing tasks. This paper presents the description of theoretical framework for investigating the types of problem posing abilities, related difficulties, their preferences of strategies and developing problem posing tasks regard to metacognitive awareness. The research instrument consists of secondary data such as the learning theories which are related to the problem posing and metacognitive machineries, in addition proposed frameworks for problem solving -posing process. The review resources reveal that problem posing tasks can provide appropriate situations for engaging students in specific learning process through inquiry -based learning environment which stress on social constructivism ideas. On the other hand, educators can establish a balance in procedural and conceptual knowledge by integrating cognitive- metacognitive strategies to this active learning. Consequently, we assert that this pedagogical perspective can encourage teachers to incorporate problem posing activities in teaching- learning materials.

Keywords: Metacognitio, mathematical problem posing, cognitive engagement, higher-order-thinking skills.

1. Introduction

The current trends of Iranian mathematics undergraduate curriculum are equipping teaching and learning materials in higher education to suitable activities that can promote advanced skills such as creative thinking, decision-making, problem solving and team working among undergraduates who will compose the voting public, the consumers, and the workforce in the near future. To achieve these goals, one of the major concerns is engaging students in mathematical problem solving situations, particularly in “real life” problems. However, the recent classroom instruction is limited to problem solving tasks alone appears ineffective in developing students’ thinking skills and higher level cognitive abilities. Therefore, mathematics teachers should complete problem solving tasks by problem posing tasks for challenging learners to the quality problems whose solution strategies are not immediately known to each student, as a result can stimulate high-order-thinking among them.

In this regard, problem posing can be applied as a means of instruction where the students pose questions in response to different circumstances, namely real life situations, another mathematical problem, or the teacher (Stoyanova, 2005). On the other hand, Students’ perception on the subject matter is profoundly altered while constructing their own original mathematical questions and subsequently observed that all their created ideas became the focus of discussion among peers. Meanwhile, the shift of responsibility problem posing from teachers to students could embed pupils in metacognitive strategies during face to face (FTF) interactions in classroom settings and led them to be independent learners(Watson & Mason, 2002).

Despite of educational researchers, over the decades, have reached to consensus about effectiveness mathematical problem posing activities, these valid tasks remain under shadow problem solving approaches in developing pedagogies. This could be due to lack of a comprehensive theoretical framework that can justify how learning can occur in problem posing environment and how these activities can implement as a operational tasks in mathematics classrooms which are limited in subject content and time . Furthermore, a cognitive-metacognitive model of problem posing process is needed to give a novel direct to class activities based on both problem solving and posing, as a result learners can equip to more competency.

This paper presents the description of theoretical framework for investigating the types of problem posing abilities, related difficulties, their preferences of strategies and developing problem posing tasks regard to metacognitive awareness. In addition, due to the importance of cognitive engagement in making classroom effective activities, it is explained how mathematical problem posing tasks can stimulate higher-order-thinking skills among undergraduate.

2. “Learning Theories” as pedagogical perspectives

"Learning Theories" are elaborate hypotheses that describe how exactly learning procedure occurs. they could take directly attention to crucial variables which have significant role in solution practical problems, additionally these theories provide appropriate methods for interpreting the examples of learning in term of conceptual framework. In this study ,learning theories as secondary data consist of social constructivism theory(Ernest, 1991),inquiry–based- learning(Bruner, 1966), face to face learning environment (Tall 1985, 2007), zon of proximal development (Vygotsky, 1978) and metacognitive theory (Flavell 2004). They specify the relationship between problem posing phenomena and the component of teaching -learning environment. Consequently, Figure.1 as a schematic diagram of the theoretical framework is drawn to illustrate how the types of problem posing abilities, related difficulties, their preferences of strategies, and levels of performances are clearly identified and defined and influenced by metacognition approaches.

3. Links social constructivism theory to Problem Posing Activities

Ernest (1991) stressed mathematics as a social construction in "social constructivism" theory, namely learning occurs during a process which the learner actively builds new ideas or concepts. Therefore, the components of this theory can justify how knowledge transfer via implementing problem posing tasks in mathematics classroom.

Linguistic knowledge, conventions and rules form the basis for mathematical knowledge, Therefore, they is necessary for understanding the given problem posing regard to situations(free, semi-structure and structure).

In addition, interpersonal social processes are needed to turn an individual’s subjective mathematical knowledge into accepted objective knowledge that is sociality understood. In the other words, constructivism’s special features are observed in the interplay between subjective and objective knowledge. In problem posing view, these elements are arisen when peers discuss together about tasks' situation (e.g. objective knowledge), then constructing their own problems (subjective knowledge). Continually , peers encounter together by posed problem(objective knowledge).In addition, objective knowledge is transferred to subjective knowledge when students is investigating solvability or insolvability generated problems. Another interpersonal social process happens in altering unsolvable problem to solvable problem (subjective knowledge) by guiding teacher. However all of the mention stages directly depend on students' prior learning experiences.

Most significant, constructivism asserts mathematical learning is a process of individual construction with sensitive to social interventions. For instance, cultural factors influence many aspects of cognitive processes that students deploy in thinking and problem solving, such as knowledge base and structural organization. In the other word, cultural differences learners may lead them to perform in different ways of problem posing namely generate problems in different ways because of their prior experience, culture, and community.

Conclusively, through glass social constructivism theory, we can monitor the types of problem posing abilities among pupils, their preferences of strategies, and levels of performances. Hence, social constructivism idea stress on problem posing tasks can provide appropriate situations for engaging students in specific learning process through "Inquiry -Based Learning" environment.

4. Problem posing activity through inquiry -based -learning

“Inquiry- Based- Learning" theory presents a more effective method of instruction based on teamwork situations. In this method, the teacher begins with a question, and then allows the students to search for information and learn on their own with the teacher's guidance. The most significant learning processes that students engage in during inquiry-learning include: creating questions of their own, obtaining supporting evidence to answer the question(s), explaining the evidence collected, connecting the explanation to the knowledge obtained from the investigative process and creating an argument and justification for the explanation. As a result, the problem solving - posing tasks can be called as beneficial educational activities if associated with “Inquiry - Based - Learning “environment.

In this respect, Bruner (1966) declared a range of philosophical, curricular and pedagogical approaches to teaching as" Inquiry -Based- Learning "theory. He emphasized that pedagogy and curriculum should require students to work together during solving problems, as a form of active learning. However, Engelbrecht, Bergsten, and Kagesten (2009) reported that mathematics teaching and learning environments are teacher-centred at majority of the universities mathematics classrooms and the most tasks and examination tests are considered procedural in character, and more formal and demand in the concepts. Therefore, "Inquiry- Based- Learning" reminds education should trend toward novel attitude that students' role is beyond problem solvers as well as they can become skilful in discovering and correctly posing problems. When students begin posing their own original mathematical questions and see these questions become the focus of discussion, their perception of the subject is profoundly altered. Meanwhile, these activities could embed them in metacognitive strategies during face to face (FTF) interactions in classroom settings, and lead them to independent learners. Consequently, teachers' role can be reformed in this environment.

Teachers should be viewed as facilitators of learning. Due to problem posing definitions as a means of instruction and activity ,a powerful tools diagnostic and a way exercise of real life situations, then these tasks in teachers 'hands not only facilitate learning process but optimize it by establishing balance between conceptual understanding and procedural understanding. Furthermore, teachers' role as guidance during the altering unsolvable posed problems to solvable can be labelled to expeditor. On the other hand, integrating problem posing activities in mathematics lessons enable teachers to know level of their students' mathematical knowledge and the ways that students can lead to understanding better of mathematics concepts. Since teachers have an crucial role in the implementation of problem posing in curriculum, they have to develop skills in problem posing and must be able to create tasks with suitable situations which engage their students in problem posing for generating a strict image of concepts and procedures for representing them in future problems(Goldenberg, 2003) .

There are three types of representation of human knowledge in mathematics, namely, "Enactive", "Iconic" and "Symbolic". In the other word, it asserts to importance integration of internalize knowledge as well as mastery in formal and precise mathematics' languages for involving in mathematical problem solving and posing correctly and deeply. Yu and Li (2008) revealed that the participants' knowledge system stored in the minds is incomplete and their internalize knowledge cannot be integrated effectively. For example, they could not know concept and understanding propositions multi-angles and multi-levels. Lowrie(2002) asserted that the children are less successful in identifying the types of understanding or processes that would be required to develop a solution when they are posing more novel problems. The lack of connection between school mathematics and real life experiences prevented children from recognizing formal symbolisms as representing verbal mathematical problems, so that children have weakness to construct a variety of problems in formal task. Abdollahpour(2011) stressed that Iranian undergraduate students have the significant difficulties related to using formal mathematics language and symbols during structural proof in "calculus 1" questions. On the other hand, overemphasizing the importance of providing a proof prevented the development of inquiry abilities. Whilst, Christou et al.(2005) found that posing problems based on given problems can be a useful strategy for found develop a new thinking situation, thinking of the given information in the problem statement, thinking of best strategy to solve it using his own questions that lead him/her to solution and thinking of more information related to the given information .

Therefore, problem posing activities can provide a suitable environment based inquiry base learning in mathematics classroom, due to teacher can shift some their responsibilities of problem posing to students who must inquiry given conditions by reviewing the part of material used for constructing a new product (Watson & Mason, 2002). Additionally , teachers can take pupils in preferred strategies for monitoring learners' level of performances , the types of their weakness in term of revised Bloom taxonomy such as understanding, applying ,evaluating and creating . These results indicate what types of supplements are needed for developing academic achievement related to problem posing tasks.

5. Role of zone of proximal development in Problem posing tasks

Vygotsky (1978), as a social constructivist, suggested that development is the product of social and cultural interaction around the shared experiences, used cognitive tools, linguistic and physical nature. On the other hand, he defined a measure called the "Zone of Proximal Development" (ZDP) as the difference between problem solving that students are capable of performing independently and their performance on problem solving with guidance or collaboration. This means that it is the range of abilities that a person can perform with assistance, but cannot yet perform independently. Meanwhile, the appropriate assistances and tools as scaffolding need to guidance learner how accomplish the new task or skill, finally the scaffolding can be removed and learners will be able to complete the task independently. Hence, mathematics activities, learning strategies, peer interaction and mastery teacher are vital parts of the learning process, so that individuals construct mathematical meaning as they participate in a variety of communities within which particular mathematical practices, reasoning, conceptions, beliefs, and interaction patterns are shared.

In this respect, problem posing activities and strategies can be considered as scaffoldings that should be structured around projects that engage students in the solution of a particular community based problem, school based problem or regional problem relevance to their worlds. Furthermore, teachers as particular scaffolding are experts who should control students' achievements in problem posing tasks and seek their difficulties in these attempts. In addition, they can lead students toward removing their weakness especially in favourite problem posing strategies.

Nevertheless , researchers stressed to importance of teachers ' role and applied strategies in enhancing pupils ' problem posing skills , a educational instruction must be able to turn learners toward independency. In the other word, it is necessary that problem posing tasks are equipped to Cognitive-metacognitive strategies which gradually lead them to independent learners to more competencies such as self- questioning, self- regulation and so on. Therefore, a communication is needed to argue how metacognitive skills can develop problem posing abilities.

6. Importance of metacognitive in problem posing tasks

Flavell (2004) defined that metacognition knowledge is one's knowledge concerning one's own cognitive processes, executive and control that can be divided as declarative, procedural and conditional. Declarative knowledge is knowledge about oneself as a learner and about what factors influence one's performance, and on the other procedural knowledge is knowledge of how to do things. Furthermore, conditional knowledge is related to know when and why to use declarative and procedural knowledge. On the other hand, ability to use the metacognitive knowledge strategically is called as metacognitive skills which generally consist of self-instruction, self-questioning and self-monitoring. In this respect, Metacognition in problem solving refers to the knowledge and processes used to guide thinking directed toward the successful resolution of a problem (McCormick, 2003). In the other word, metacognitive skills support problem solvers in understanding the problem, selecting suitable solution strategies, monitoring solution strategies effectively, and identifying and overcoming obstacles to solving the problems. Ultimately, metacognition is an important component for incorporating appropriate information and strategies during the problem solving.

Accordingly, metacognitive strategies are a type of scaffolding that can help the student to improve his/her problem solving and posing skills. Scaffolding can take many forms, for example, cueing or the posing of metacognitive questions (e.g., "What is your goal?" and "What strategy do you use?"). Kapa (2001) found that when students were cued during at ask they became more successful in problem solving activities than students who were cued only afterwards. Therefore, when teachers use these metacogintion questions in a continually bases, students will get access to the development of their metacognitive strategies and self-guiding in their learning processes, ultimately become independent learners. On the other hand, there exist scaffolding for developing metacognition within the framework of constructivism learning, such as implementing problem posing strategies in mathematics classroom that can encourage students to ask effective questions in term of metacognitive abilities.

Due to importance these phenomena in effective learning, mathematics experts investigated metacognitive abilities in problem solving process, whilst the types of these abilities in problem posing activities are partly identified. In this respect , Md Nor and Ilfi (2012) reported that the phases of metacognition involved in problem posing tasks among secondary school students are reading, planning, interpreting, and checking which adopted from Phang's(2009) categories. The metacognitive skills involved in mathematical problem posing tasks are yet to be determined among undergraduates as there are several sets of metacognitive skills engaged in problem solving suggested by pervious researchers (Thomas, 2012). Additionally, some communications is needed to indicate how implementing problem posing and metacognition instruments can promote mathematical thinking (Tall, 2007) and high - order - thinking in mathematics classroom interaction.

7. Face to Face learning environment

"Didactic triangle" model related to face to face learning environment with the multiple relations among the three vertices namely, the student, the teacher and the mathematics were presented by Tall (1985). In a practical problem posing activities, students are active learners who should be applied mathematics teaching - learning material for fostering mathematical thinking by teacher's guidance.

Tall (2007) categorized mathematical thinking into three significantly worlds namely, conceptual-embodied world, perceptual-symbolic world, and axiomatic-formal world. The theory of three worlds of mathematical thinking provides a rich structure in which to understand and interpret mathematical learning and thinking at all levels:

- The conceptual-embodied world based on our physical perceptions that are built into mental conceptions through reflection and thought experiment.
- The perceptual-symbolic world that begins with real-world actions (e.g. differentiation, integration) and symbolization into concepts (e.g. derivative, integral) developing symbolic that operate both as processes (e.g. differentiation, integration) to do and concepts (e.g. derivative, integral) to think about (called precept).
- The axiomatic-formal world based on formal definitions and proof.

Mahir (2009) investigated how conceptual and procedural are involved in variety subjects of calculus, continually, she found that participants' conceptual understanding is lower than their procedural understanding , namely they have been unbalanced by trainings technique-centred. In mathematics education, conceptual understanding is knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics as well as procedural mathematics understanding focuses on skills and step-by-step procedures without explicit reference to mathematical idea. Meanwhile, pure procedural skills without awareness of concepts underlying them often are unsuccessful in gaining readily appropriate methods to solve mathematics problems, in contrast, students that have conceptual understanding also have successful procedural performance. Consequently, mathematics teaching should aim to establish balance between both procedural and conceptual knowledge parallel foster metacognitive skills among undergraduates; as a result, they can be guided toward upper stage of Bloom's taxonomy which associated with higher-level thinking.

8. Conclusion

This paper presents pedagogical perspectives on problem posing and metacognition approaches by learning theories which can direct educational experts' attitude toward designing the effectiveness means of instruction. This study asserts that cultural differences and prior educational experience influence strictly learners' mathematical problem posing abilities, their performances and preferences problem posing strategies. Consequently, experts should be intended the social aspects and curriculum contents that a community has been engaged in them to each of mathematical problem posing activities. Furthermore, the review resources reveal that problem posing tasks can provide appropriate situations for involving students in specific learning process through inquiry-based learning environment which stress on social constructivism ideas for transferring knowledge via interaction between objective and subjective knowledge. On the other hand, it clarifies the role of teachers as guidance as well as mathematical problem posing tasks, their strategies and metacognitive skills as the vital scaffolds for developing students' performance along mathematics problem posing attempts. As a result, these interventions during face to face (FTF) interactions in classroom settings led gradually pupils to be independent learners with a balance among the conceptual-embodied, the perceptual-symbolic and axiomatic-formal. Briefly, Figure.1 presents a theoretical framework of links between considered learning theories and desired components in problem posing tasks regard to a "didactic triangle".

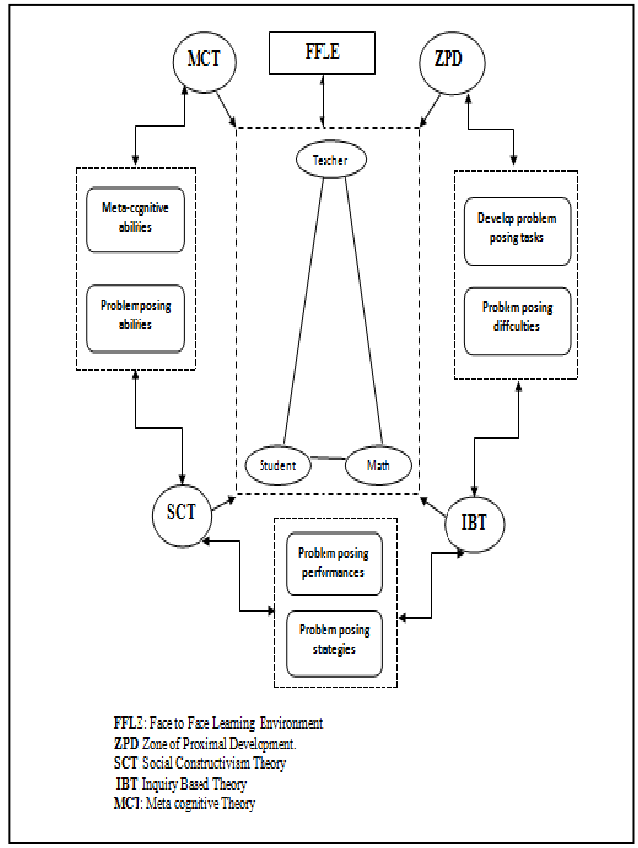

Figure1. Theoretical framework of this study.

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