



Significance Test Based on Log-Logistic Distribution

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Abstract: This paper deals with the log-logistic distribution ($\beta = 3$) as a life time model. Moments of order statistics and an ordered sample are used to define a test statistic for the null hypothesis that the considered random variable has log-logistic distribution ($\beta = 3$). The percentiles of the test statistic are evaluated, Power's of the test with half-logistic distribution and Rayleigh distribution as alternatives are also evaluated.

Keywords: Testing, Likelihood ratio criterion, Moments, Order statistics, Power of test.

1. INTRODUCTION

The well known log logistic distribution in statistics is studied by many researchers with respect to various problems on statistical inference. To make the distribution more applicable to the data on non-negative values, a suitable transformation of the logistic variate may be considered analogous to similar transformations on a normal variate. In this direction a transformation of the logistic variate is considered to develop a new distribution known as log-logistic distribution with positive space for the random variable (Shah and Dave[19]), Malik[16], Block &Rao[5], Tadikamalla and Johnson[23]). Order statistics from log-logistic distribution with their properties, estimation based on order statistics are developed by Ragab and Green ([17], [18]). Two stage estimation for log-logistic model was developed by SrinivasaRao and Kantam[21].

The probability density function of log-logistic distribution is given by

$$f(x) = \frac{\beta x^{\beta-1}}{(1+x^\beta)^2}, x \geq 0, \beta > 1$$
$$= 0, \text{ otherwise}$$

1.1

It can be seen that log-logistic distribution is increasing failure rate (IFR), constant failure rate (CFR) or decreasing failure rate (DFR) model which is much useful in life testing and reliability studies. If a scale parameter is introduced in the above distribution, it is called scaled log-logistic distribution and it is given by

$$f_{x/\sigma}(x) = \frac{\beta}{\sigma} \frac{\left(\frac{x}{\sigma}\right)^{\beta-1}}{\left(1 + \left(\frac{x}{\sigma}\right)^\beta\right)^2}, x > 0, \quad \sigma > 0, \quad \beta > 1$$

= 0, otherwise

1.2

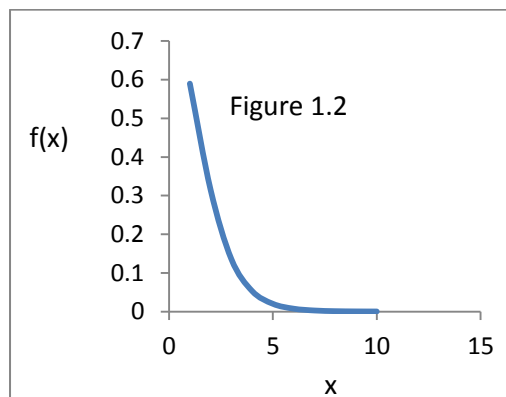
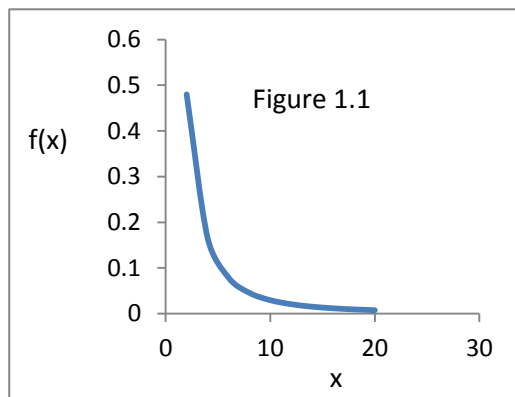
2. THE GRAPHS OF FREQUENCY CURVE OF LOG-LOGISTIC DISTRIBUTION (BETA =3)

The graphs of frequency curve of log-logistic distribution ($\beta=3$) for certain parametric combinations look similar to half-logistic, Rayleigh models. Incidentally, log-logistic distribution ($\beta=3$) is a combination of half-logistic distribution

and Rayleigh distribution. We present below the frequency curve of log-logistic distribution ($\beta=3$) that look similar to half-logistic distribution, Rayleigh distribution for a parametric combination.

LLD

HLD



RAYLEIGH DISTRIBUTION

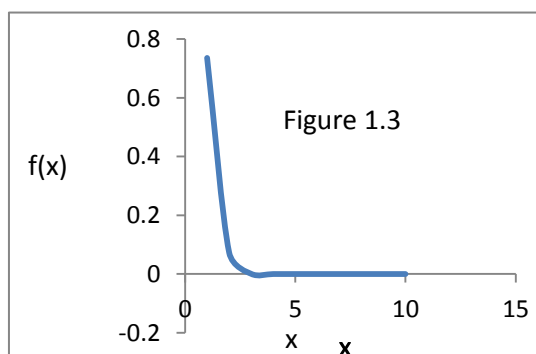


Figure 1

We therefore are interested to study the discrimination between for log-logistic distribution ($\beta=3$) and half-logistic, Rayleigh distribution at this parametric combination. Another objective of this study is to know whether half-logistic distribution or Rayleigh distribution is a reasonable or better alternative to log-logistic distribution in order to adopt the available simpler and admissible inferential procedures of half-logistic distribution, Rayleigh to log-logistic distribution data. Such studies of discriminatory problems between probability models are made by Gupta *et al*[6], Gupta and Kundu[7], Gupta and Kundu[8], Kundu and Gupta [9], Kundu and Manglick [12], Kundu *et al* [10], Kundu and Manglick [11], Kundu [13] and [15], Kundu and Ragab [14], Arabin and Kundu [1], and Kundu [2], SrinivasaRao and Kantam [20] studies the significance test for the half-logistic distribution. Arabin and Kundu [3], Arabin and Kundu[4], and the references there in.

Sultan [22] developed a test criterion to distinguish generalized exponential distribution from Weibull, normal distributions, moments of order statistics in samples drawn from generalized exponential distribution. In this paper we adopt the criterion suggested by Sultan [22] to distinguish between log-logistic distribution and half-logistic, Rayleigh distribution. A brief description of procedure developed by Sultan [22] and its applications to our model is presented in Section 2. The methodology of arriving at the critical values and powers of the test procedures are given in Section 2 respectively. In all these cases, the percentiles of respective test statistic and powers of test procedures for selected parametric conditions and sample sizes evaluated numerically are tabulated in the respective situations. Log-logistic distribution ($\beta=3$) is considered as null population (P_0) half-logistic distribution, Rayleigh distribution is considered as alternative populations (P_1).



3. TESTING OF HYPOTHESIS

Let us assume that the distribution of the life of a product is the scaled log-logistic distribution where probability density function and cumulative distribution function are respectively given by

$$f_{X/\sigma}(x) = \frac{\beta}{\sigma} \frac{\left(\frac{x}{\sigma}\right)^{\beta-1}}{\left(1+\left(\frac{x}{\sigma}\right)^\beta\right)^2}, x > 0, \sigma > 0, \beta > 1 \quad = 0, otherwise \quad 3.1$$

$$F_{X/\sigma}(x) = \frac{\left(\frac{x}{\sigma}\right)^\beta}{1 + \left(\frac{x}{\sigma}\right)^\beta}, x > 0, \sigma > 0, \beta > 1$$

$$= 0, otherwise \quad 3.2$$

where β is known shape parameter.

The probability density function of standard log-logistic distribution is given by

$$f(z) = \frac{\beta z^{\beta-1}}{(1+z^\beta)^2}, x \geq 0, \beta > 1$$

$$= 0, otherwise \quad 3.3$$

and its cumulative distribution function is given by

$$F(z) = \frac{z^\beta}{1+z^\beta}, x \geq 0, \beta > 1 \quad = 0, otherwise \quad 3.4$$

The probability density function of scaled half-logistic distribution is given by

$$f_{X/\sigma}(x) = \frac{2e^{-x/\sigma}}{(1+e^{-x/\sigma})^2}; x \geq 0$$

$$= 0, otherwise \quad 3.5$$

and its cumulative distribution function is given by

$$F_{X/\sigma}(x) = \frac{1 - e^{-x/\sigma}}{1 + e^{-x/\sigma}}; x \geq 0$$

$$= 0, otherwise \quad 3.6$$

The probability density function of standard half-logistic distribution is given by

$$f(z) = \frac{2e^{-z}}{(1+e^{-z})^2}; x \geq 0$$

$$= 0, otherwise \quad 3.7$$

and its cumulative distribution function is given by

$$F(z) = \frac{1 - e^{-z}}{1 + e^{-z}}; x \geq 0$$

$$= 0, otherwise \quad 3.8$$

The probability density function of scaled Rayleigh distribution is given by

$$f_{X/\sigma}(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}; x \geq 0$$

$$= 0, otherwise \quad 3.9$$

where $\sigma > 0$, is the scale parameter of the distribution.

$$F_{X/\sigma}(x) = 1 - e^{-x^2/2\sigma^2}; for x \in [0, \infty)$$

$$= 0, otherwise \quad 3.10$$

The probability density function of standard Rayleigh distribution is given by

$$f(z) = \frac{z}{\sigma} e^{-z^2/2}; x \geq 0$$

$$= 0, \text{ otherwise} \tag{3.11}$$

where $\sigma > 0$, is the scale parameter of the distribution.

$$F(z) = 1 - e^{-z^2/2}; \text{ for } x \in [0, \infty)$$

$$= 0, \text{ otherwise} \tag{3.12}$$

Let X_1, X_2, \dots, X_n be a random sample of size n , Here we test the null hypothesis.

P_0 : The sample has come from log-logistic distribution ($\beta=3$)

against each of the alternative hypothesis separately with distinct critical values and powers.

- (i) P_1 : The sample has come from half logistic distribution.
- (ii) P_1 : The sample has come from Rayleigh distribution.

As we mentioned in Section 1 that log logistic distribution ($\beta=3$) is a combination of half- logistic distribution or Rayleigh distribution. Hence we have considered log- logistic distribution ($\beta=3$) as null population and half-logistic distribution, Rayleigh distribution as separate alternative populations. Sultan [22] suggested a test statistic given by the formula

$$T = \frac{\sum_{i=1}^n x_{(i)} \alpha_{(i)}}{\sqrt{\sum_{i=1}^n x_{(i)}^2 \sum_{i=1}^n \alpha_{(i)}^2}} \tag{3.13}$$

where $x_{(i)}$ is the order observation in the sample, $\alpha_{(i)}$ is the expected value of i^{th} standard order statistic in a sample of size n from the null population. If x_1, x_2, \dots, x_n is a sample of size n from the null population the Equation (3.13) for T is used as a test statistic to discriminate a null population and the corresponding alternative population with the help of its critical values. Hence, the sampling distribution of T and its percentiles therefore are essential to make use of the test statistic T . Since, sample quantiles are consistent estimators of population quantiles, we propose a statistic similar to T based on population quantiles rather than moments of order statistics and is given by

$$T^* = \frac{\sum_{i=1}^n \delta_{(i)} x_{(i)}}{\sqrt{\sum_{i=1}^n x_{(i)}^2 \sum_{i=1}^n \delta_{(i)}^2}} \tag{3.14}$$

$$\text{where } \delta_{(i)} = F^{-1}(p) = \left(\frac{p_i}{1-p_i}\right)^{\frac{1}{\beta}} \text{ with } p_i = \frac{i}{n+1}$$

We have tabulated the percentiles of empirical sampling distribution of T^* $n=5(5)25$ with ($\beta=3$) through 10,000 Monte-Carlo simulation runs and are given in Table 2.2. The percentiles of T^* would serve as critical values to test null hypothesis that a given sample comes from log-logistic distribution ($\beta=3$). Therefore a large value of T^* implies a strong linear relation between $x_{(i)}$ and $\delta_{(i)}$, which means that the sample is taken from Log- logistic distribution ($\beta=3$). On the other hand, a small value of T^* indicates that the sample is from a distribution different from log- logistic distribution ($\beta=3$).

The decision to reject the null hypothesis is based on the critical values of T with a pre-assigned level of significance. Accordingly, the percentiles of T^* are essential to carry out the test procedure regarding the above hypothesis. For example, let t_α denote the α - percentile of T^* , then if

$$P_T = \left\{ \left(\frac{t}{t} > t_\alpha \right) \right\} = 1 - \alpha \tag{3.15}$$

Holds, the null hypothesis cannot be rejected with level α . Thus, the percentiles of T^* are essential, but unfortunately it is not possible to obtain the distribution of T^* analytically. We determined the percentiles of T^* by means of simulation on the basis of 10,000 simulation runs for various sample sizes. The obtained percentiles are given in Table 3.2.

4. POWER OF THE TEST

The power of a test refers to the probability of not rejecting a wrong null hypothesis and this probability depends on the assumed alternative distribution. Here in this case, the null hypothesis given by log-logistic distribution ($\beta=3$), as the alternative distribution we consider half- logistic distribution. The test statistic T^* is calculated by generating a sample of a specified size from each of the alternative populations with the α_i in the formula for T^* being those of log-



logistic distribution ($\beta=3$). The corresponding calculated value of T^* is compared with the corresponding critical value given in Table 2.1. This process is repeated 10,000 times and the number of rejections of null hypothesis of the null hypothesis out of 10,000 times is taken as a measure of power of the test i.e.,

$$\text{power of the test} = \frac{\text{number of rejections of } H_0}{10,000} \tag{4.1}$$

The calculations of the power were performed for the above mentioned alternative with various sample sizes and various levels of significance for complete samples were presented in Tables 4.1 and 4.2 and the tables indicate the high power of the proposed test for all considered alternatives. The power indicates that the half-logistic alternative is more powerful and Rayleigh is less sensitive as alternative population. This shows that the test statistics clearly discriminates a LLD ($\beta = 3$) from half-logistic and Rayleigh with decreasing power in that order.

Table 3.1 Percentiles of T^* for complete samples

n \ p	5	10	15	20	25
0.00135	0.775183	0.701203	0.704152	0.665101	0.651014
0.005	0.808711	0.773299	0.763001	0.751439	0.740614
0.01	0.840312	0.808237	0.808775	0.804359	0.780282
0.025	0.881919	0.862865	0.86271	0.864364	0.864891
0.05	0.913615	0.905506	0.903768	0.908602	0.909287
0.10	0.942904	0.942379	0.943865	0.945402	0.947113
0.90	0.996428	0.996134	0.996416	0.996692	0.996905
0.95	0.997562	0.997178	0.997322	0.997515	0.997628
0.975	0.998246	0.997797	0.997923	0.998026	0.998113
0.99	0.998876	0.998334	0.998390	0.998485	0.998506
0.995	0.999210	0.99869	0.998671	0.998723	0.998803
0.99865	0.999604	0.999004	0.998941	0.999031	0.998985

Table 3.2 Estimates of the power of test for two alternative distributions

n	Half-logistic distribution			Rayleigh distribution		
	0.99	0.95	0.90	0.99	0.95	0.90
5	0.9972	0.9872	0.9732	0.9894	0.9504	0.9001
10	0.9988	0.9961	0.9892	0.9911	0.9559	0.9025
15	0.9997	0.9984	0.9955	0.9946	0.9607	0.9156
20	0.9998	0.9992	0.9973	0.9945	0.9962	0.9230
25	1.0000	0.9998	0.9985	0.9944	0.9712	0.9329

Table 4.1 Power T values for LLD (3) vs HLD

S.No.	Sample no	99%	95%	90%
1	5	0.9972	0.9872	0.9732
2	10	0.9988	0.9961	0.9892
3	15	0.9997	0.9984	0.9955
4	20	0.9998	0.9992	0.9973
5	25	1.0000	0.9998	0.9985

Table 4.2 Power T values for LLD (3) vs Rayleigh

S.No.	Sample no	99%	99.865%	95%	90%	13.5%
1	5	0.9894	0.9983	0.9504	0.9001	0
2	10	0.9911	0.9987	0.9559	0.9025	0
3	15	0.9946	0.9985	0.9607	0.9156	0
4	20	0.9945	0.9995	0.9662	0.9230	0
5	25	0.9944	0.9988	0.9712	0.9329	0
6	30	0.9964	0.9997	0.9771	0.9500	0

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