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A Framework for Textbook Analysis

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Abstract: Textbooks are widely accepted as a common feature of classrooms worldwide and are important vehicles for the promotion of curricula. Consequently their content and structure are very important for the promotion of a specific vision of curriculum. There are many features of textbooks, some which go unknown to the authors, which have a significant impact on their target audience. Such features can have positive or negative impacts on learning. Textbook analysis is a means by which these features can be identified and hence the effectiveness of textbooks be established. The author in her research on mathematics textbooks has established a framework for textbook analysis based on the work of Halliday (1973), Morgan (2004), the TIMSS study (Valverde et al., (2002)) and Rivers (1990) which comprises four key elements; Content, Structure, Expectation and Language. The author is hence using her own research on mathematics textbooks to develop and highlight aspects of textbook analysis.
Textbook analysis is particularly important to support educational reform and hence this chapter sets out to establish the significance of conducting textbook research and highlighting best practice in the area.

Keywords: Textbook analysis, Theoretical frameworks, Textbook Language Analysis

1.1 Introduction

It is accepted worldwide that mathematics and science textbooks have a major influence on classroom practice (Valverde et al., 2002). Textbooks are important tools for the promotion of specific types of curricula. They are organised in a purposeful way, and consequently their content and structure are very important for the promotion of a specific vision of a curriculum.

Over the past twenty years changes in practice and teaching methodologies have led to concerns regarding the quality of mathematics and science textbooks. Robitaille and Travers (1992) express the view that textbook content and how such textbooks are used impact directly on students' learning. While it is widely accepted that the curriculum is central to influencing the choice and treatment of subject matter in mathematics classrooms, one of the key factors in implementing this content is the textbook (Schmidt, McKnight, Valverde, Houang, and Wiley, 1997). Many researchers (Skemp, 1982; Van Dormolen, 1986; Pimm, 1997; Hiebert and Carpenter, 1992; Dowling, 1996; Orton, 2004) have looked specifically at some of the key concerns with mathematics textbooks; however, few researchers have examined the textbook as a whole with the exception of the Third International Mathematics and Science Study (TIMSS) (Valverde et al., 2002).

The author’s main focus and research to date is primarily concerned with the analysis of mathematics textbooks. The Irish Mathematics Education system is currently in the midst of rolling out a new mathematics curriculum entitled Project Maths. Project Maths differs greatly from the old curriculum in terms of intention, the aim of Project Maths is to move away from didactical teaching styles and teaching to the exam and instead focus on teaching and learning for understanding. Hence, the introduction of this new curriculum has placed a spotlight of many features of Irish mathematics classrooms, one such feature being the mathematics textbook. Hence the author’s work on mathematics textbooks to date can inform educators and policy makers alike on the quality of the mathematics textbooks.

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1.2 The Role of Textbooks in Education

> *Textbooks are artefacts. They are a part of schooling that many stakeholders have the chance to examine and understand (or misunderstand). In most classrooms they are the physical tools most intimately connected to teaching and learning. Textbooks are designed to translate the abstractions of curriculum policy into operations that teachers and students can carry out. They are intended as mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms. Their precise mediating role may vary according to the specifics of different nations, educational systems and classrooms. Their great importance is constant.*
>
> (Valverde et al., 2002: 2)

According to both Ravitch (2003) and Valverde et al., (2002) textbooks are vitally important, they play a significant role in shaping teachers', students' and families' views of school subjects. Textbooks can be defined simply as books which are written for the purpose of teaching and/or learning. However, as noted by Venezky (1992: 437) "from a single set of curriculum guidelines an infinite number of textbooks could be built, each with its own interpretation of the intent of the guidelines". Textbooks are the closest thing students have to working from the curriculum and the purpose of these textbooks is to assist with student learning. Despite such an obvious relationship between the textbook and the student there is limited evidence which outlines how students actually use their textbooks. In the context of education in general and mathematics education in particular research which highlights textbook use is limited to how teachers use their textbooks. Textbooks are a vital ingredient of successful learning. The importance of their role can never be exaggerated. Mathematics teachers have been found to rely on textbooks for at least 90% of their teaching time (Mikk, 2000). Such a statistic can only highlight the need for good textbooks.

The role of the textbook varies greatly from classroom to classroom and teacher to teacher, however Gelfman, Podstrigich, and Losinskaya (2004) provide a basic outline for the intermediary role of the textbook:

- To teach and encourage students to construct new knowledge,
- To balance detail and precision of information,
- To provide logical and consistent mathematical systems,
- To bring about new questions,
- To provide students with active, creative, many sided information.

While Sewall (1992) goes so far as to say that it is almost impossible to achieve a high level of education without the use of textbooks. According to Valverde and Schmidt (1998) the major failing of textbooks occurs when teachers try to cover every aspect of it, hindering or ignoring the application of suitable methodologies for teaching and learning where necessary. Horsley and Laws (1992) claim that notion of teachers not using textbooks effectively cannot be correct if there are good textbooks in place. The purpose of the textbook is to help and motivate students to learn. Mikk (2000: 17) highlights the need for exciting, imaginative textbooks; "students have many sources of information available, if their textbooks are dull, they are unwilling to study them. Interesting and enthusiastic textbooks develop curiosity and interest in the subject".

In conjunction with is central role in the classroom the textbook also carries the burden of the role of link between intended and implemented curriculum. Curriculum is a central variable in TIMSS and is used to compare national systems of education. The conceptual framework for TIMSS is based on the now well-known tripartite model of curriculum (Robitaille et al., 1997):

- intended curriculum
- implemented curriculum
- attained curriculum.

Valverde et al., (2002) regard the textbook as the potentially implemented curriculum (Figure 1) forging a direct link between intended and implemented curricula.

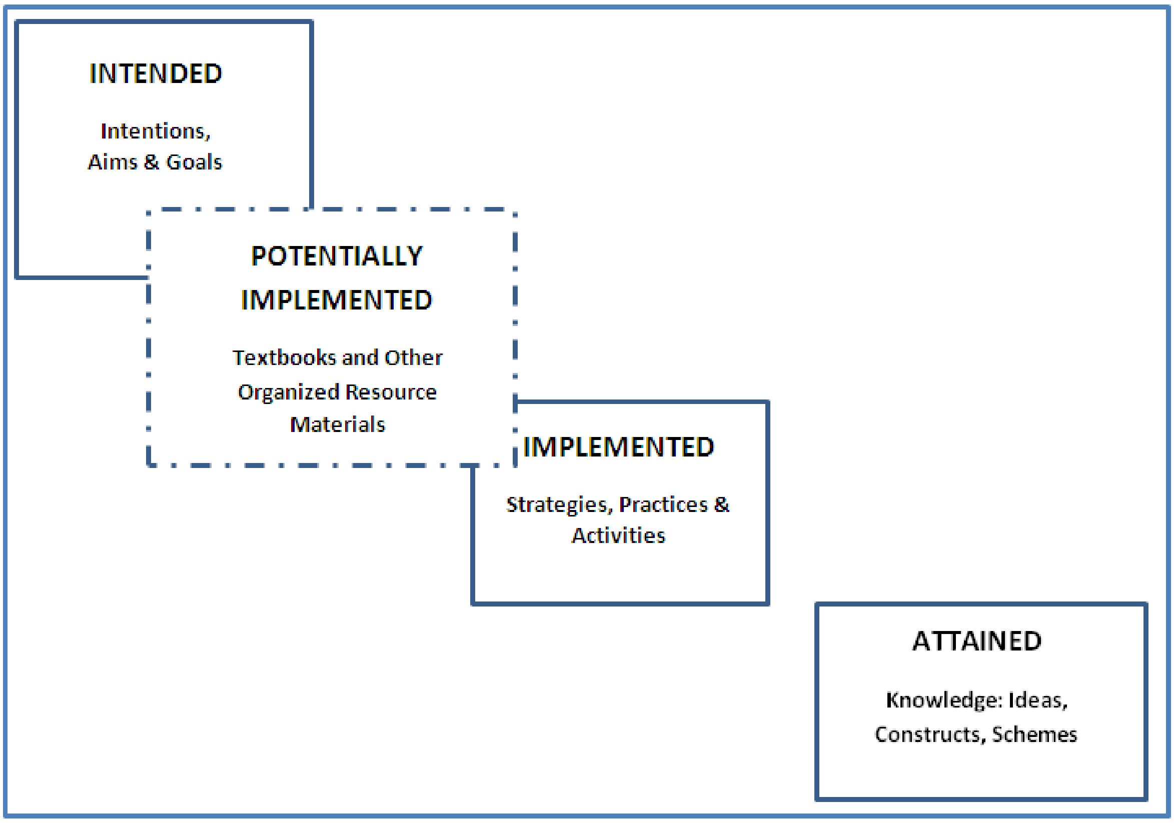

Figure 1: Textbooks and the Tripartite Model (Valverde et al. 2002: 13)

In Figure 1, the *intended* curriculum is formed by the educational system and national policy, comprising "content standards, curriculum guides, frameworks or other such documents" (Valverde et al., 2002: 9). The *implemented curriculum* is created by a combination of classroom practice and the teacher. Valverde et al., (2002) create a powerful link between the intended and the implemented curricula in their creation of the *potentially implemented* curriculum, affected primarily by the textbook. Hence, TIMSS posits and develops a powerful link between curriculum and textbooks, which they strengthen by describing the textbook as a 'surrogate curriculum'. This point is expanded by Venezky (cited in Robitaille et al., 1997: 50) when he points out that a single set of curriculum guidelines can spawn a myriad of textbook representations. In this context the bi-directional link between the textbook and curriculum is a powerful insight.

1.3 The Importance of Textbook Analysis

The practice of using textbooks is, according to Robinson (1981), as old as the practice of writing. The word textbook appeared in the 1830's long after 'collocation' textbook (Love and Pimm, 1996). According to Walbesser (1973) the first arithmetic textbook was written by Isaac Greenwood in 1729; `Arithmetick, Vulgar and Decimal'. The sequencing of this textbook was as follows:

- Present a rule,
- Provide example which uses the rule,
- Exercises for students to apply the rule,
- Formal proof of the rule.

While Walbesser was referring to the situation present in the seventeen hundreds, the reality is that little has changed. This process of “rule - example – practice is still with us today" (Walbesser, 1973: 63). Lockhart (2002: 16) quotes Bertrand Russell when he speaks of how little change is evident in mathematics,

> “I was made learn by heart: `The square of the sum of two numbers is equal to the sum of their squares increased by twice their product' - I had not the vaguest idea what this meant and when I could not remember the words, my tutor threw the book at my head, which did not stimulate my intellect in any way".

Walbesser also informs of the indifference of publishers with regard to visual appeal of the textbooks, it was 1834 before drawings appeared in American mathematics textbooks. Walbesser noted an emphasis on the practice of rule-example-practice in his research more than thirty years ago and despite his research being dated little has changed with regard the process of rule-example-practice or in the development of textbook appeal.

The educational value of a textbook is crucially important. According the Mikk, (2000: 77) “Textbook analysis dates back to 900 AD when Talmudists counted words and ideas in texts". A textbook is something which students will be reading on a daily basis and any messages, no matter how small or innocent, are open for interpretation by a young impressionable mind. The process of developing, editing and publishing of a textbook should incorporate a number of people. Mikk (2000) suggests that this `working team' should comprise a subject specialist, a teacher, an education psychologist, an illustrator and a text specialist in order for all considerations to be embraced. Good textbooks need to consider content, value forming aspects, motivational elements, accessibility, illustrations, study guides etc.; they must encourage a thirst for knowledge. Vygotsky (1956, cited in Mikk, 2000: 69) talks about the `zone of proximal development'. He identifies the need for textbooks to direct students to such a zone, one where there is optimal learning. Difficult tasks cause frustration and tasks which are considered too easy have little influence on students' progress. There is a need for textbooks to encourage students to work in this zone. Research can help ensure textbooks are focused on reaching such a goal.

In the Cockroft Report (Cockroft, 1982), there is reference to the increase in popularity of a `Problem Solving' approach in the early 1980's, however, Cockroft notes that despite the change in curriculum focus and the obvious acceptance of this approach, textbooks never followed suit and as such the textbooks were one of the main factors contributing to its failure in that they failed to adapt accordingly. Since the Cockroft Report many curriculum initiatives have applied a problem solving approach or an adapted problem solving approach. One such initiative is currently underway in Ireland at present. Project Maths, a new mathematics curriculum which was implemented nationwide in September 2010, incorporates many of the ideals of the problem solving approach such as focusing on problem solving strategies more so than emphasising the memorisation of rules and procedures. In a recent report conducted by O’Keeffe and O’Donoghue (2011b) they noted that historically, mathematics teaching and classrooms in Ireland have been strongly influenced by commercially produced school textbooks that have promoted a view of mathematics concerned mainly with skills and instrumental learning (NCCA, 2005). This view of mathematics curriculum is not compatible with the new curriculum initiative Project Maths. Hence, mathematics textbook analysis is essential to ascertain the view of mathematics that is portrayed by the newly designed mathematics textbooks. If such didactical emphases dominate through the new generation of mathematics textbooks then the success of Project Maths is likely to be severely compromised. Hence textbook analysis can support the development and success of evolving curricula and new teaching and learning initiatives.

1.4 The TIMSS Study – Mathematics and Science Textbook Analysis

The Third International Mathematics and Science Study (1995) (TIMSS) provides the foundation for much research on mathematics and science textbook analysis. TIMSS devised a common framework to compare systems of education through analyses of curricula, related documents and artefacts. They are known as *curriculum frameworks*. Each framework is characterised by the same three elements that are further sub-divided (Robitaille et al., 1997):

- subject matter content
- performance expectations
- perspectives or context.

These frameworks are applied to the curriculum or any piece of the curriculum that is seen as promoting the intended, implemented or attained curriculum and includes artefacts such as textbooks, curriculum guides, standards documents etc. TIMSS employs two separate frameworks viz. the curriculum framework for mathematics, and curriculum framework for science. The TIMSS model was formulated to deal with evolving curricula.

1.5 Initial development of a framework for Mathematics Textbook Analysis

In much of her own work on mathematics textbook analysis the author looks to the Third International Mathematics and Science Study (1995) (TIMSS) for its theoretical underpinnings. The mathematics framework is a tool for studying curriculum or any piece of curriculum or artefact. Indeed the view supported by TIMSS is that 'A textbook is a surrogate curriculum…' (Robitaille et al., 1997: 50). Hence, in her study the author adapted the TIMSS framework for mathematics curriculum analysis for use as a tool for mathematics textbook analysis (O'Keeffe, 2011a).

Thus we start with the TIMSS framework for mathematics curriculum analysis as a tool for textbook analysis. It has three dimensions:

- Structure
- Performance expectations
- Perspectives.

Subsequently, the mathematics framework was adapted and refined for use in TIMSS as an instrument for mathematics textbook analysis per se.

Hence, the TIMSS mathematics curriculum framework as it evolved is adapted and further refined as outlined below for the author's own research. The 'perspectives' dimension captures student data and is not used in the author's textbook analysis study. In any case it was not envisaged that all three dimensions would be applied to every piece of curriculum. The structure dimension encompasses issues concerning content and the structure of knowledge and information in the textbook and the make-up of the textbook. This line of reasoning led to an analytical tool with two dimensions and three elements as follows:

- Structure
  - Structure
  - Content
- Expectation.

Further refinements were added to this TIMSS instrument by O'Keeffe (2011a) in order to allow for a finer-grained analysis. Refinements based on the work of River's (1990) and Mikk (2000) that reinforce and add to the TIMSS model around content and expectation and structure analysis respectively, are included here. The initial evolved model comprises three key elements (Figure 2).

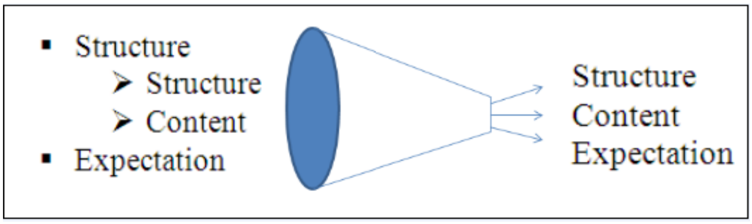

Figure 2: Initial Development of the framework for Mathematics Textbook Analysis (O'Keeffe, 2011b)

1.5.1 Structure Analysis

Textbook structure adds to or takes from textbook comprehension suggesting that succession and connections between text elements need to be analysed carefully. Halliday and Hasan (1993 cited in Mikk, 2000: 94) broke text cohesion/structure down into 5 parts; reference (pronominal, comparatives, articles), substitution (nominal, clausal, etc.), ellipsis (nominal, verbal, etc.), conjunction (additive, temporal etc.), lexical (same item, general item, etc.). Mikk (2000: 99) illustrates by way of a matrix table how one can easily analyse the structure of a text and record diagrammatically how frequent ideas/topics appear and therefore connections are visualised. In order for a structure to impact positively there are a number of key issues which need to be incorporated. These were analysed in the TIMSS report as `Physical Scale'. While the structure of the knowledge within a textbook is vital the physical structure will determine whether the intended audience will even consider the text. It includes many aspects such as those outlined by Valverde et al. (2002); Area and framing, Elements (pictorial, verbal, design), Colour and Non colour, Information levels, Unification and Separation.

1.5.2 Content Analysis

Textbook content influences the selections and emphases applied by teachers and students, consequently impacting on learning outcomes (Mulryan, 1984). Rivers (1990) discusses four aspects of content analysis which are similar to those outlined by Gerbner (1969). She created four subheadings in the area of content analysis:

- Motivational factors - which includes historical notes, scientist and mathematician biographies, career information, applications and photographs,
- Comprehension cues - focuses on colour and graphics,
- Technical Aids - includes all material related to calculators and computers,
- Philosophical Position - emphasis and predominant philosophy.

These subheadings are easily identified for analysis and their role in effective teaching is transparent. Wittlin (1978), whose work on museum exhibits was connected with science textbook analysis by Robinson (1981), insists that the very first objective of any textbook must be to attract student attention. Then the focus switches to presenting the message clearly and comprehensibly, and finally maintaining attention (Robinson, 1981). The following table outlines Wittlin’s recommendations, which need to be considered in content analysis especially with regard to motivational factors. According to Wittlin (1978) one must first attract attention (Initial Arousal), then present a clear and concise message (Attending Message reception) and finally maintain attention (Maintenance of Attention).

Table 1: Factors to Consider for Textbook Analysis, Wittlin (1978)

Initial Arousal	Attending Message Reception	Maintenance of Attention
Danger: Underestimation	**Danger:** Overestimation	**Danger:** Monotony
To Avoid the Danger: Relevance, Interest, Dissonance, Sensory, Appeal, Appeal to effect	**To Avoid the Danger:** White Space, Signal Noise, Planned Redundancy, Integration of multiple channels, Hierarchal organisation under key areas	**To Avoid the Danger:** Change modality, Insert questions , Vary senses used, Drama of Issues

1.5.3 Expectation Analysis

Expectation is the third element of the TIMSS framework for textbook analysis. Performance expectations are embedded throughout textbooks and will impact significantly on how students choose to deal with the topics presented. For example if the focus of a mathematics textbook is on repetition and practice then a student will subconsciously look to replicate a previous method as soon as he/she encounters a question, without attempting to use any problem solving skills. The most basic consideration of expectation is that students and teachers alike will read and understand the material presented (Valverde et al., 2002). The TIMSS Report (Valverde et al., 2002) identified expectation as crucial to textbook analysis and identified 19 different expectations that can be placed on students throughout textbook chapters. The Rivers Matrix (Rivers, 1990) also contains an expectation component. It looks to examine the presence of emphases and philosophies throughout textbooks. Both the emphasis and philosophy put forward by a textbook have a direct bearing on student expectations.

1.6 A framework for Mathematics Textbook Analysis

The author extends and develops her framework for mathematics textbooks analysis (identified in Figure 2) by applying an analytical tool adapted from the language analysis of written texts (student's own written texts), functional grammar analysis (Morgan, 2004). The language analysis tool enables the researcher to develop an overall view of the textbook in terms of student learning while also allowing for a better understanding of the difficulty of the mathematical language as encountered by students using textbooks.

1.6.1 Language

Students should be able to communicate mathematics, both `verbally and in written form' (NCTM, 1989). However, it seems that students are expected to acquire this communicative ability by osmosis. That is, they must acquire it themselves from textbooks or notes. Also, much of the notation and symbols used in these textbooks may not be conducive to learning. Mulryan (1984) describes primary textbooks as having an excessive vocabulary load, variability of word meaning, insufficient repetition of mathematical terms and inadequate vocabulary control. How to use mathematical language is not something that is taught in Irish mathematics education, yet the significance of the language of mathematics to learning is acknowledged. When analysing mathematical textbook language Newall (1990) found a number of features in textbook language such as discourse type (narration, description etc.), coordinators (connectors between sentences) and semantic structures. These features can provide a basis to inform textbook analysis of mathematical language. Mulryan (1984) provides three subheadings for language, analysis; word signifiers, notational signs and graphical signs. These areas can be analysed, with consideration of Newall's language features, under the following headings:

- Word signifiers:
    - General vocabulary: word signs used regularly in daily life e.g. and, from.
    - Mathematical terms: term with specific mathematical meaning, there are two types technical or special.
    - Technical vocabulary: word signs peculiar to math e.g. Heptagon, multiple.
    - Special vocabulary: word signs used in daily life which have different mathematical meaning e.g. match, set, group or figure.
    - Abbreviations: shortened or abbreviated technical words such as cm, km, HCF etc.
    - Letters: alphabetical letters which represent numbers, lines.
- Notational signs:
    - Notation signs: Hindu- Arabic number systems or signs such as >.
- Graphical signs:
    - Pictorial/diagrams symbols: pictures/graphs which demonstrate mathematical principles.

The most significant feature of language is of course the words used. In a study carried out by Marks et al. (1974), they replaced 15% of the words in a text with more commonly used words and presented the text to 600 6th grade students. They found that comprehension was increased from 47% to 73%. Word length also has a significant impact on student learning, the longer the word the more information there is contained in it, making it more difficult to fully understand. Mathematical terms need to be explained, and their meanings need to be understood by the reader. The development of thinking can be divided into three stages, active, figurative and abstract. This would suggest that when learning/teaching the focus should be on being actively involved and engaged in ideas and where possible physical objects. The visual centre in our brains is approximately thirty times bigger than the audio centre hence it is often easier to understand something which is visual.

Sentence complexity is known as syntactic complicacy and involves sentences and paragraphs. To comprehend a sentence one must first remember it, long sentences cannot be remembered easily thus making them complicated. Luria (1975) analysed sentence complexity and found that the following causes confusion; inversion (a later event being mentioned before earlier one for example if a textbook mentions simultaneous equations with three unknowns before simultaneous with two unknowns), multi-meaningful phrases, subordinate connections, distant constructions, triplet comparisons and double negation. A study carried out by Glynn and Britton (1986) focused on analysing frequency of words, sentence length, study aims, emphasising headings, questions for actualising and prior knowledge. They found that all of the above played a vital role in students' ability to acquire the knowledge, their time spent reading the text and the mental effort it took for them to do so. Another study carried out by Klare (1963) found that suitable readability levels proved to increase effectiveness of text in over 68% of cases they investigated.

1.6.2 Readability

The term readability refers to a number of factors which influence the reader, including interest and motivation, legibility of the print, complexity of the words and sentences in relation to the ability of the reader. Interest and motivation are especially significant for a textbook. John Holt, quoted by Mann (1981) defines a textbook as “a book that no-one would read unless they had to". This idea is reinforced by Wiest (2003) who highlights the significance of reader interest to readability levels of a textbook, the more interest the book can evoke from the reader the deeper the level of comprehension and understanding attained. Wiest also talks about including novel or demanding stimuli in favour of simple stimuli since engaging students in fantasy demands higher comprehension levels. Davy 1987 (cited in Mikk, 2000: 79) noted that textbooks with familiar words are easier to understand. Mikk (2000: 79) used an electro-oculograph to fix the eye movements of students while reading a passage and found that they spent more `time and fixations' on unfamiliar words. Word frequency assessment (Cloze Test) is the most common method for assessing familiarity of words. This can then create a frequency dictionary, for the students in the subject area, to create a list of commonly used words. Wiio (1968, cited by Mikk (2000: 81)) devised a modification ratio which can be used as an indication of text complicacy. Many formulae such as this have been created and modified over the years, and it is recommended that to ensure an accurate result more than one formula should be applied to your text.
There are many varieties and adaptations of readability formulae, the most commonly used formulae are the Flesch Reading Ease and Flesch - Kincaid Grade level (TxReadability, 1998).
Flesch Reading Ease

- The Flesch Reading Ease gives an output from 1 - 100. The higher the output the easier a text is to read.
- The Flesch Reading Ease Formula = 206.835 - (1.015 _ ASL) - (84.6 _ ASW).
    - Where ASL is the average sentence length, i.e. the number of words in the whole text divided by the number of sentences.
    - ASW is the average number of syllables per word, i.e. the syllable count for the whole text divided by the word count for whole text.

Flesch - Kincaid Grade level

- The Flesch -Kincaid Grade level gives a grade readability result. The output value will indicate which grade level the text is most suitable for.
- The Flesch -Kincaid Grade Formula = (0.39 _ ASL) + (11.8 _ ASW) - 15.59.
  - Where ASL is the average sentence length, i.e. the number of words in the whole text divided by the number of sentences.
  - ASW is the average number of syllables per word, i.e. the syllable count for the whole text divided by the word count for whole text.

However, these readability tests are designed to analyse English-language paragraphs. Much mathematics research, which involves readability measurement, uses the above tests as a basis of comparison of readability levels but none of these readability tests can effectively and accurately measure the actual readability of a mathematical text. Mathematical texts as previously stated combine ordinary English with mathematical English and symbols (Taylor and Hargreaves, 1999). This varied information embedded in mathematical text is unlikely, according to Thomas (1997), to ever be fully understood by such English-language based readability tests. This raises the issue for a need to create a mathematically focused test for readability. Many studies have indicated the obvious lack of correlation between standard readability scores, problem solving performance and comprehension (Paul, Nibbelink, and Hiiver, 1986; Hembree, 1992; Wiest, 1967). Despite the direct connection between readability and problem solving (a highly topical element in many countries including Ireland), “research in reading and mathematics continues to attract little attention" (Thomas, 1997: 39).

1.6.3 A framework for Mathematics Textbook Analysis

Researchers have identified that mathematics is a complicated, diverse but unique language. Bullock (1994) reinforced the significance of this language when highlighting the fact that Newton had to invent calculus in order to develop and express his ideas and in 1987 Pimm compared the learning of mathematics to the learning of a foreign language. Language analysis and its significance has been widely researched for a number of years and has formed a significant part of mathematical research from the early 1990's, with, for example, the work of Halliday (1973); Skemp (1982); Van Dormolen (1986); Pimm (1987); Noonan (1990); Chapman (1993); Dowling (1996); Mikk (2000); Morgan (2004); Orton (2004), with Mikk and Morgan focusing particularly on the role of language in mathematics texts or textbooks. For the purpose of mathematics textbook analysis the author draws primarily upon the work of Halliday (1973) and Morgan (2004). Halliday's research provides the basis for much language analysis in many different subject areas, focusing on the functional aspects of language. He outlines this functional aspect as the way in which language is used, the purpose that it serves and the way in which a reader can achieve these purposes. One of the reasons Halliday outlines for following this line of investigation is to \establish general principles relating to the use of language". For this reason Halliday's functional grammar analysis is applicable to this study, as the author is seeking to not only analyse the language present in mathematical text but also to research the overall effectiveness of the language for teaching and learning. Halliday's functional grammar analysis is based on three elements:

- Ideational Function,
- Interpersonal Function,
- Textual Function.

Halliday developed his functional grammar analysis for language in general and the language of mathematics never featured as a standalone unit within his work. However, in 2004 Morgan applied this functional grammar analysis to mathematics texts. Morgan (2004) applied Halliday's functional grammar analysis framework to students' own written mathematical texts. She describes how the ideational function can look specifically at the mathematics and the mathematical activities presented while the interpersonal function highlights sources of concern in the mathematics language such as the use of the word “we"

(identified by Pimm (1987). as a cause for concern). The textual function identifies the formation of argument in a mathematical text and any message portrayed via reports, descriptions or narratives.
Morgan's framework (2004) for mathematics language analysis can be applied effectively to any mathematical text. The framework originated with the work of Halliday (1973) and is not mathematically specific, however, Morgan herself used this framework for analysing students' own mathematical writings and now the author has applied it to school mathematics textbooks. Morgan's framework, while applied in its entirety, differs from the analysis in this study with regards to the interpretation element. The function of the language analysis tool is to work in conjunction with the other elements of textbook analysis to provide an overall view of the impact of the mathematics textbook on student learning. Thus providing a framework for Mathematics Textbook Analysis with four key elements (the theoretical underpinnings of which are provided in Table 2);

- Structure,
- Content,
- Expectation,
- Language.

Table 2: Theoretical Frameworks Supporting Mathematics Textbook Analysis

Theoretical Frameworks	Function	Significance
TIMSS **(Valverde et al., 2002)**	Provides overall structure for textbook analysis	**Significant to three key elements:** 1. Content Analysis 2. Structure Analysis 3. Expectation Analysis
Rivers (1990)	Reinforces the TIMSS framework and adds new dimension	**Significant to two key elements:** 1. Content Analysis 2. Expectation Analysis
Mikk (2000)	Reinforces the TIMSS framework and adds new dimension	**Significant to one key elements:** 1. Structure Analysis
Morgan (2004)	Strengthens textbook analysis by providing a fourth element for analysis	**Significant to one key elements:** 2. Language Analysis

TIMSS (2002)
The most well-known, international textbook study was conducted by TIMSS and reported by Valverde et al. (2002). TIMSS involved an in-depth analysis of 630 mathematics and science textbooks. The TIMSS analysis comprised of three key elements; Content, Structure and Expectation. In order to effectively and systematically complete research on mathematics textbook analysis the TIMSS method of both analysis and presentation provides many insights.

Rivers (1990)
In 1990 Janelle Rivers undertook a two-part analysis of first year algebra textbooks in South Carolina. Phase 1 of her study focused on a comparison of five textbooks, and in-depth analysis of the:

- Motivational Factors,
- Comprehension Cues,
- Technical Aids,
- Philosophical Orientation.

She also looked at the cost and sales figures for each textbook. Part 2 of her study was based on the changes made to each textbook based on the NCTM 1 standards. Part 1 of the Rivers study is directly applicable to this research study with the first three of the Rivers elements (listed above) overlapping and reinforcing the data analysis and collection of the TIMSS content analysis. The fourth and final element supports the TIMSS expectation analysis.

Mikk (2000)
Mikk (2000) encompasses many issues and concerns of mathematics textbook analysis related to the use, evaluation and analysis of textbooks. In his research Mikk illustrates by way of a matrix table how one can easily analyse the structure of a text and record diagrammatically how frequent ideas/topics appear and there- fore connections are visualised (Appendix D) (Mikk, 2000: 99). This method of structure analysis is combined with that of TIMSS to strengthen the data collection for structure analysis.

Morgan (2004)
Having spent a number of years as a mathematics teacher Morgan completed her PhD in 1995 which focused on the analysis of discourse of written mathematical reports. Following on from here Morgan has continued her research in the area of language and mathematics. One of the key areas emerging from Morgan's work is the analysis of written mathematical text; however she focuses on student's own mathematical writings. Her work is primarily based on Halliday's functional grammar analysis. This framework, created by Halliday (1973) which was applied to mathematics by Morgan (2004), is utilised in this research study to analyse the language of the mathematics textbooks.

1.7 Conclusion

It is likely that the central role of textbooks in the classroom will continue despite the emphasis on e-learning and e-learning objects and curriculum initiates such as that underway in Ireland at present. While the ‘textbook’ as we know it may change, the pedagogical considerations of the ‘text material’ will still focus on enhancing and supporting teaching and learning and hence the core values of the traditional textbook will remain important. Studies such as TIMSS have demonstrated the practical importance of curriculum materials, namely the textbook, in mathematics and science teaching and learning. Mathematics teaching and learning are both impacted by the quality of the mathematics textbooks that are available. Therefore research in the area of mathematics textbook analysis is worthy of consideration.

The TIMSS model for curriculum analysis provides much of the theoretical underpinnings required to effectively analyse mathematics and science textbooks. However, it is important to note that while the TIMSS framework does provide a robust model for mathematics and science textbook analysis which allow for evolving curricula ensuring its applicability into the future, it is not without shortcomings. The author’s research (O’Keeffe, 2011a) has identified some areas for inclusion such as analysis of textbook language. The language of mathematics is an essential part of mathematics and how it is presented and developed within mathematics textbook can impact on student learning. Hence, it stands to reason that a framework for textbook analysis should consider the overall impact of the language in the textbook. The framework for mathematics textbook analysis as presented by the author is built on the TIMSS model but provides a further adaption which contributes to improving the analysis of the textbook as a ‘whole’. The author’s framework for mathematics textbook analysis comprises of four key areas; Content, Structure, Expectation and Language each of which emerged from the literature as contributing to the overall effectiveness of a textbook.

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