Study of The Frequency Characteristics of a Ring Oscillator

Arya Tah[1], Debasish Kumar Rakshit[2]

[1,2]*Department of Electronics and Communications, NIT Durgapur, India*
[1]*Email Address: arya1910tah@gmail.com*
[2]*Email Address: debasishmodern100@gmail.com*

Abstract: In the upcoming article, we lay down a particular method of devising the frequency parameter of ring oscillators. We also bring about a comparative analysis of ring oscillators, and try to find out a way of relating their frequency characteristics with their inherent resistance and capacitance. The formula derived here is far more simplified as compared to all the others present. Moreover, it allows quick, precise efficient formulation of frequency without much pain. Also a detailed analysis of CMOS and DTMOS based design has been carried out and a parameter has been devised to determine the efficiency of both CMOS and DTMOS. The simulation platform used here is LTSpiceIV and the used models are based on BSIM4.0 level 54 CMOS technology.

Keywords: delay, ring oscillator, inverter, power dissipation, frequency, Elmore delay calculation.

I. INTRODUCTION

The importance of Ring Oscillators in the electronics industry is unquestionable. With the immense and fast developments in the field of VLSI, their importance too has increased; given the fact that it is an integral part of most electronic systems. With the advent of ADCs, PLLs and VCOs which have tremendous usage of ring oscillators, it has become a necessity to formulate and study their frequency characteristics of ring oscillators in a precise and lucid fashion. Ring oscillators comprise of a ring of N-stages of inverters, where N is necessarily odd and the output oscillates between two limits, HIGH and LOW. We know that frequency of such oscillation can be increased by decreasing the number of stages or by altering device dimensions, which often end up disastrously increasing the power consumption. Now, we are required to estimate the frequency of operation of the oscillator. The inverter based delay, td, can be found out, and hence by conventional methods we can find out the ring oscillator frequency, by the age old formula $f = 1/2Nt_d$.

However, the delay evaluation of each inverter stage remains a huge problem. This requires quite an amount of knowledge about internal device parameters. There have been methods, where the estimation formulae have been derived without much use of device parameters but, in those cases the efficiency of the formula is not appreciable. There have been a large number of works, but all of them have their own handicaps, in being applicable only to a particular device length or device (W/L) ratio. Also, there are quite a number of formulae which do not actively work beyond CMOS technology. The proposed method largely oversees

these deficits and is applicable not only at any (W/L) ratio, but also, even on changing device characteristics this formulation holds true.

The formula, prescribed here depends on only 3 parameters, namely the number of stages, the capacitance at each stage, and the resistance, which in turn is estimated by Barkhausen's criterion and power dissipation at a single stage.

II. DEVELOPMENT OF THE PROPOSED TECHNIQUE

2.1 Study of existing formulae and making of a new estimation process

The most basic ring oscillator is simply a chain of single ended digital inverters, with the output of the last stage fed back to the input of the first stage. Note that to provide the DC inversion, an odd number of stages must be used. To see why this circuit will oscillate, assume that the output of the first inverter is a '0'. Therefore, the output of the N-th inverter, where N is odd, must also be'0'.However, this output is also the input to the first inverter, so the first inverter's output must switch to a'1'.By the same logic, the output of the last inverter will eventually switch to a '1', switching the output of the first inverter back to '0'. This process will repeat indefinitely, resulting in the voltage at each node oscillating.

Let's assume that the delay each inverter gives is t_d. So, the net delay associated with N stages will be $N*t_d*2$. This is because t_d is nothing but the difference in time of the toggle points of the input and corresponding output.

We know the net phase shift should be of the order of 2π. But the net phase shift per oscillator must be of the order of π/N; as the remaining shift is obtained by the DC inversion. Now, this is one of the basic requirements that would come into play while we calculate the delay.

Now, our work involves studying of frequency response in case of an N-stage oscillator. We have seen already that frequency depends on t_d. We can say that every stage in the ring oscillator has a resistance and capacitance associated with it, which ultimately causes the delay. Now, to start with we use inverter stages using 50nm CMOS technology, with W_{pmos}=1000nm, and W_{nmos}=500nm.

Table 1. Studying The Effects of Adding Identical Capacitive Load at The Terminus of Each Stage.

S.No.	DELAY TABLE		
	External load	*n*	*delay*
1.	0pf	5	0.22ns
2.	0.01pf	5	0.44ns
3.	0.02pf	5	0.67ns
4.	0pf	7	0.32ns
5.	0.01pf	7	0.64ns
6.	0.02pf	7	0.97ns
7.	0pf	9	0.42ns
8.	0.01pf	9	0.84ns

In the table above we have seen the effect of adding load capacitances at the terminal of each stage of the ring oscillator. Now, adding capacitance would increase delay, as we have seen t_d is proportional to Capacitance [3]. We have to analyze the dynamic operation of the inverter in this context.

Let us now analyze the results in the table. We know that, every ring oscillator has its characteristic capacitance. If we equal the load capacitance to the intrinsic capacitance that each stage of the oscillator provides, we find that the frequency halves. Thus, we may conclude, that when frequency halves, our estimated capacitance becomes equal to the intrinsic capacitance.

2.2 Studying the effect of capacitance on frequency

From [5] we get to know about Miller's effect and about replacing bridging capacitances with a single capacitance. In this way we bring about replacement in the transistors used in this context, and thus we get an equivalent load C_l across the inverter stage. Thus when we add another $C=C_l$ at the terminal of each stage, the equivalent load becomes $2C_l$.

Now, there are quite a number of ways to find C_l. Using the SPICE model files and hence device parameters for capacitance estimation is often tedious and time consuming. Also, the evaluation becomes a lot more difficult for advanced CMOS technology models.

So, a better way is experimental determination, by way of plots. The table below, Table 2 gives us the delay in relation to $(W/L)_p$,$(W/L)_n$ and $(W/L)_p+(W/L)_n$ ratio.

Table 2. Study of capacitance with respect to W/l Ratios.

S.No.	CAPACITANCE TABLE		
	$(W/L)_p$	$(W/L)_n$	C
1.	(1000/50)=20	(500/50)=10	0.010pf
2.	(1000/50)=20	(1000/50)=20	0.012pf
3.	(1250/50)=25	(500/50)=10	0.011pf
4.	(1000/50)=20	(3000/50)=60	0.023pf
5.	(4000/50)=80	(3000/50)=60	0.039pf

Capacitance can also be calculated analytically, by way of formulae given in [6]. Model manuals give us detailed description about how to calculate capacitance, and capacitance at the terminal of each stage can be calculated by Miller's effect.

However, as expected, the experimental results would vastly be equal to the results obtained by model files.

Table 3. Study Of Capacitance With $(w/l)_p+(w/l)_n$.

S.No.	CAPACITANCE TABLE		
	$(W/L)_p,(W/L)_n$	$(W/L)_p+(W/L)_n$	C
1.	20,10	30	0.010pf
2.	25,10	35	0.011pf
3.	20,20	40	0.012pf
4.	20,60	80	0.023pf
5.	80,60	140	0.039pf

An observation can be accredited from the above table; capacitance is doubling with doubling (w/l) ratios. So, we can make a conclusion, by further close observation, on Table 3 that

C_1/C_2 is approximately equal to (sum of (W/L) ratios of p and n transistors)$_1$/ (sum of (W/L) ratios of p and n transistors)$_2$. This is a rough way of estimating the capacitance at each stage of the ring oscillator.

The delay is therefore proportional to N (number of stages) and C (capacitance).

Also, there must be a resistance term to equate the left and right sides. Hence, there must be a R. Our next objective is to determine the Resistance, R.

The gate drain overlap capacitance of Q_1, C_{gd1} is replaced by an equivalent capacitance between output node and ground of $2C_{gd1}$. Each of the drain body capacitances has a terminal at a constant voltage. Thus, these are replaced by equal capacitances between output node and ground. Since the second inverter does not switch states we will assume that the input capacitances of the upcoming stage remain constant and equal to gate capacitance.

$$C=2C_{gd1}+2C_{gd2}+C_{db1}+C_{db2}+C_{g3}+C_{g4}+C_w.[7].$$

2.3 Study of resistance parameter and estimating resistance

Now, the important thing is estimating the resistance parameter. We have found out a new experimental way, of finding out resistance using power dissipation at a single stage. A point to be noted here is the power dissipation of a single stage is same as long as the device parameters remain the same. However, it does not depend on the length of the ring oscillator in any sense.

If the ring oscillator stages are replaced by their linear equivalents, then the whole loop can be reconstructed as shown in Fig.7.

The important thing that is done here is frequency domain analysis of the loop. The loop gain X(s) =A (s1).A (s2)…A (sn).More often than not, A (s1) = A (s2) =…= A (sn). [8,9]

First, let's start analyzing the Barkhausen's criterion. The Barkhausen's criterion states that the net phase of the loop gain should be zero. Given that should happen, the net phase shift needs to be $2k\pi$. However, π phase shift is obtained from DC inversion. Hence the phase shift of the rest of the loop has to be π. Hence, we find phase shift per stage of the oscillator is π/N. The general practice however remains, lessening the required phase shift, hence to minimize the number of required stages.

We can obviously state at this juncture that the total phase shift of RC delay $=\pm\pi$.

Thus if phase shift per stage is Θ then:

$$\Theta =\pm\pi/N \tag{1}$$

The model proposed here, hence would have,

$$\tan^{-1}(RC\omega) =\Theta \tag{[8](2)}$$

Hence, our oscillation frequency becomes:

$$\omega = (\tan\Theta)/RC \tag{3}$$

We know that, for an inverter in operation, in each cycle, the energy dissipation in the Qn and Qp is $0.5CV_{DD}^2$ respectively. Hence the net energy dissipation is equal to CV_{DD}^2. Now, we would like to frame a possible evaluation of power.

$$P= CV_{DD}^2 /T \tag{4}$$

$$P=f\ C\ CV_{DD}^2 \tag{5}$$

$$f=P/ CV_{DD}^2 \tag{6}$$

$$\omega= P.2\pi/ CV_{DD}^2 \tag{7}$$

Hence, now for finding a possible estimate of resistance we equate equation 3 and 7.

$$(\tan\Theta)/RC= P.2\pi/ CV_{DD}^2 \tag{8}$$

$$R= V_{DD}^2 (\tan\Theta)/2\ \pi P \tag{9}$$

For our case, VDD has always been 1V, so our result would well be

$$R= (\tan\Theta)/2\ \pi P \tag{10}$$

This is how we have developed an effective and efficient way of Resistance estimation.

However in reference [9] they have taken

$\Theta =2k\pi/N$, which is certainly not the case as the remaining phase shift is provided by DC inversion.

The advantages of using this method are many. Firstly we do not have to engage ourselves into tedious calculations. Next, by knowing power dissipation of a single stage, of a particular design of inverter, the resistance can be calculated for any length of ring oscillator, i.e. having any number of permissible stages.

With this, we approach the crucial juncture, where estimation of capacitances and resistances has been done.

So, we see, the 3 parameters on which the frequency depends, namely N,R,C have been sorted out to the best possible extent. The next task is to frame a relation between the parameters.

2.4 Framing a relation between number of stages ,frequency, resistance, capacitance

Frequency of operation is the parameter of interest for us. But, we know, f=1/T. We can say, T is proportional to N, R, and C as we can see. What we are actually interested in is nothing but the propagation delay, that would ultimate give us the frequency. [4]

Quick delay estimation is the core basis of designing faster and critical paths. We may use the simulator based approach for our delay and hence frequency estimation purpose. But that does not help our understanding of paths in any case, hence faster; more efficient pathways cannot be developed. Novice designers, spend hours tweaking parameters to find out efficient model designs.

In this section we take up a lumped circuit RC model of transistors. Although transistors have complex, nonlinear current voltage characteristics, they can be well approximated as a switch in series with a resistor, where the *effective resistance* would depend on the average current delivered by the transistor. Transistor gate and diffusion nodes have capacitance.

So, we in this arena, devise a method, where the net delay measurement would be on the basis of driving resistance and load capacitance. Usually in the technology, we use devices of minimal length, and optimum power consumption.

We take up the Elmore delay model for our purpose of synthesizing the delay of the ring oscillator.

Viewing ON transistors as resistors, a possible way of viewing the circuit is as shown in Figure 7. Elmore Delay states that the net delay over the whole network is equivalent to the summation of the product of individual load capacitance at each node and the subsequent resistance between Node and Source.

$$T = \sum_{k=0}^{n} R_k C_k \tag{11}$$

Now, let's take up the case of investigating how to implement the technique of Elmore delay to Ring oscillator.

Ring oscillator does not contain any source as such, from where we would calculate the resistance per node. So, we take up an analogous method. We know, capacitors can act as voltage sources, when they are charged. Thus, for our sake, we take the ring oscillator to be composed of (N-1) +1 delay stages. So, for our analysis, there are N-1 stages of resistors and capacitors. The remaining stage is termed the 'Engine stage'. We, take up that it is this capacitor-resistor stage that acts as voltage source.

As ring oscillators have connected ends, we can take this 'Engine stage' anywhere in the ring.

A better representation would involve, cutting the ring at the Kth node. (K≤N). So the (K+1)th node ((K+1) th stage of capacitance and resistance) is termed the engine node (engine stage). The remaining length of the ring is analyzed by Elmore delay analysis:

There would be (N-1) stages of identical capacitance and resistance.

In case of added capacitive load, we take the capacitance as (C_p+C_l), where C_p is the intrinsic capacitance of the transistor and C_l is the added capacitive load at each level.

So our formula becomes:

$$T_{pd} = \sum_{k=0}^{n-1} R_k C_k \qquad (12)$$

For no added capacitive load, for a N stage ring oscillator, this formula becomes:

$$T_{pd} = N(N-1)RC/2 \qquad (13)$$

(It is because, T_{pd} becomes RC + 2RC+...+ (N-1)RC, and the driving or engine stage is excluded as it acts as the source).

So, a method has been devised to obtain both driving resistance and delay associated with a ring oscillator. Resistance has been calculated by the method given above. While delay estimation, is nothing but the summation of the product of estimated resistance and capacitance over (N-1) nodes. Now, it's time to check whether our proposal holds experimentally by simulation, and thus bring about a detailed study about accuracy of the proposed method.

III. EXPERIMENTATIONS AND CHECKING OF THE PROPOSED METHOD

Now, we will check whether our proposed method holds by using simulation tool LTspiceIV and BSIM 4.0 model using 50nm CMOS technology. First we have studied ring oscillators using 50nm CMOS technology with width of NMOS being 500nm and width of PMOS being 1000nm. All the stages have a capacitance of 0.01pf as has been shown in a table above.[1]

Table 4.   Readings For A Ring Oscillator Using BSIM 4.0 Model With 50nm CMOS Technology.

S.No.	DELAY TABLE				
	n	*power*	*RC(in 10^{-11}s)*	T_d*spice*	T_d*calcu.*
1.	3	53.12 uW	5.161	0.13 ns	0.13 ns
2.	5	53.12 uW	2.167	0.21 ns	0.21 ns
3.	7	53.12 uW	1.661	0.30 ns	0.297 ns
4.	9	53.12 uW	1.090	0.41 ns	0.42 ns
5.	11	53.12 uW	0.876	0.46 ns	0.47 ns
6.	13	53.12 uW	0.73	0.56 ns	0.567 ns
7.	15	53.12 uW	0.63	0.69 ns	0.67 ns

So, we find that our formulation does predict results with great accuracy as far as working with a W_p=1um, W_n=500nm, $L_{p,n}$=50nm based CMOS based ring oscillator. Now, we find out, whether our proposed method holds true for any generalized case, i.e. for any $(W/L)_{p,n}$ ratio. The table below shows the results of simulation using transistors of different (W/L) ratio.

Table 5. Frequency response when the (W/l) Ratios are different.

S.No.	DELAY TABLE					
	n	*power*	W_p	W_n	T_d*spice*	T_d*calcu.*
1.	5	67.12uW	1u	1u	0.21ns	0.19ns
2.	5	60.14uW	1.25u	0.5u	0.24ns	0.22ns
3.	5	95.40uW	1u	3u	0.31ns	0.28ns
4.	5	240.6uW	4u	3u	0.22ns	0.19ns
5.	5	53.11uW	1u	0.5u	0.21ns	0.21ns

Now, we see our formula operates with quite a high rate of precision for any value of (W/L) and N, for CMOS inverter stages. Now, if we can show, that this formula can operate beyond CMOS technology, the formula would encompass a huge working domain. For our purpose we would use DTMOS [1].

IV. COMPARISION OF RESULTS FOR DTMOS AND CMOS BASED RING OSCILLATORS

So, we now begin our analysis of the formula, based on DTMOS technology. DTMOS Inverter and ring oscillator have been shown in Figure 1 and Figure 2. CMOS inverter and ring oscillator have been shown in Figure 4 and Figure 5. Simulation results of both have been shown in Fig. 3 and Fig. 6. Using LTspiceIV simulator we have been able to get the following results. Model used is the same level 54 based BSIM 4.0 model. $L_n = L_p = 50$nm.

Table 6. Readings Based On DTMOS Technology Based Ring Oscillators.

S.No.	DELAY TABLE						
	n	*C(pF)*	W_p	W_n	T_d*spice*	T_d*calcu.*	*Power*
1.	5	0.065	4u	3u	0.18ns	0.18ns	396.107uW
2.	5	0.035	4u	0.5u	0.30ns	0.29ns	133.83uW
3.	7	0.013	1u	0.5u	0.26ns	0.25ns	80.533uW
4.	5	0.031	2u	2u	0.17ns	0.16ns	223.14uW
5.	5	0.02	2u	0.5u	0.22ns	0.21ns	107.68uW

So, we see our formula fits quite well for DTMOS based technology as well. So, there can't be much questionability regarding the domain of the proposed method. What we can hence see, is that our formula works well for CMOS as well as DTMOS technologies. So, that generalizes the fact that this formula is well acceptable irrespective of the technology, irrespective of the device parameters. The above quoted

technology further promises the fact of generalization because the ring oscillators can drive a higher frequency than CMOS based ones, although the power dissipation is a bit more than CMOS based ring oscillators.

V.   POWER AND FREQUENCY CHARACTERISTICS ANALYSIS

Here we would carry out a brief discussion on power and frequency characteristics of the CMOS and DTMOS inverter based ring oscillators. The power consumption is quite an important parameter as far as low power functioning of ring oscillators is concerned. Here let's take a look back at Table 5 and Table 6. Lets concentrate on the CMOS and DTMOS ring oscillator models models having length 50nm and their W_p= 4u; W_n= 3u. The power dissipation of CMOS based ring oscillator has been only 240.6 uW compared to the 396 uW required for driving the DTMOS based ring oscillator. However the delay associated with the DTMOS structures is 0.19ns compared to 0.22 ns by the CMOS ring oscillator.

   Another example that we can take is by referring to Table 4 and Table 6. We take up the case of CMOS and DTMOS models having length 50nm and W_p= 1u and W_n= 0.5u. We see, that for N=7, the delay associated with DTMOS is 0.26ns while for CMOS is 0.30 ns. Once again however we find that the DTMOS has more power consumption than CMOS model.

Thus we can hereby say, that the ring oscillator designed using CMOS technology can be used for low power requirement purposes, whereby, even at a reduced speed we would get enhanced performance. However for high speed domains, DTMOS can replace CMOS. We next define a parameter to find out the effectiveness of a particular model.

   We hereby incorporate a parameter ε where we will define this parameter as

$$\varepsilon = (1/\text{Delay})*(1/\text{Power})*10^{15}$$

Now, let us calculate the ε for both CMOS and DTMOS for N=7, W_p=1u, W_n= 0.5u, l_p=l_n=50nm.

$$\varepsilon(\text{CMOS}) = 0.06275$$

$$\varepsilon(\text{DTMOS}) = 0.04775$$

Now, if we calculate the same for CMOS and DTMOS for N=5, W_p=4u, W_n=3u, l_p=l_n=50nm

$$\varepsilon(\text{CMOS}) = 0.01889$$

$$\varepsilon(\text{DTMOS}) = 0.0140$$

Thus, we can say from the above data that for any value of (W/L) the efficiency of CMOS technology based ring oscillators are more than DTMOS based ring oscillators. But we may still shift to DTMOS based technology where and when there is no restriction based on power and the ring oscillator requires a higher frequency drivability.

CONCLUSION

In the underlying work, we thus have procured a method of evaluating the frequency that not only is suitable for a wide range of device parameters but works well outside the domain of the common CMOS technology. The formula shown here works better than all other known frequency estimation methods, and hence can be well used for frequency estimation in industries. Also, we bring about a comparison of efficiencies of CMOS and DTMOS based technologies and find out that CMOS has a better functioning quotient than DTMOS. A detailed analysis has been done, which proves the following, frequency of a ring oscillator is easily derivable, CMOS has greater effectivity than DTMOS as per our parameter, and in cases of better frequency drive DTMOS is more acceptable an option.
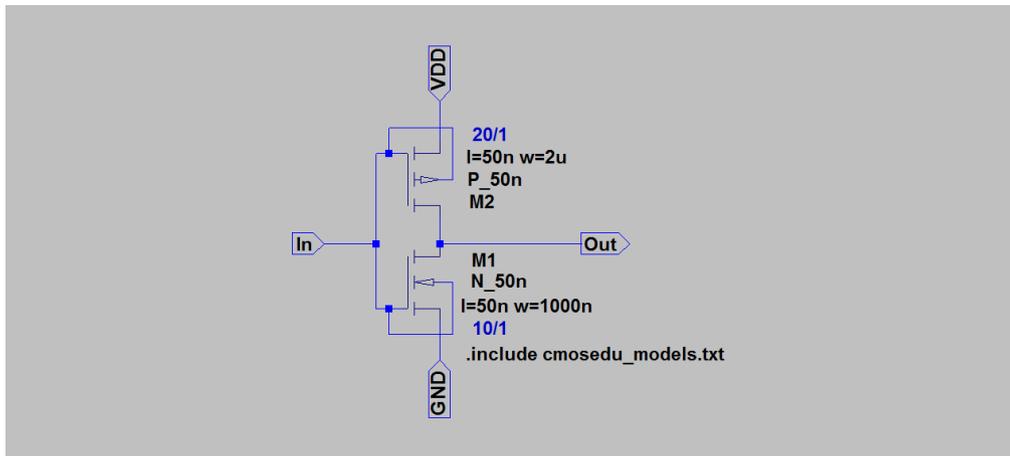
Figure 1. DTMOS inverter.
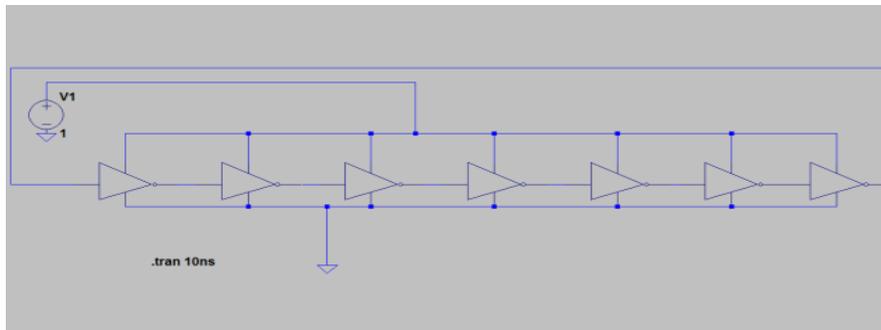
Figure 2. Ring oscillator using above inverter.
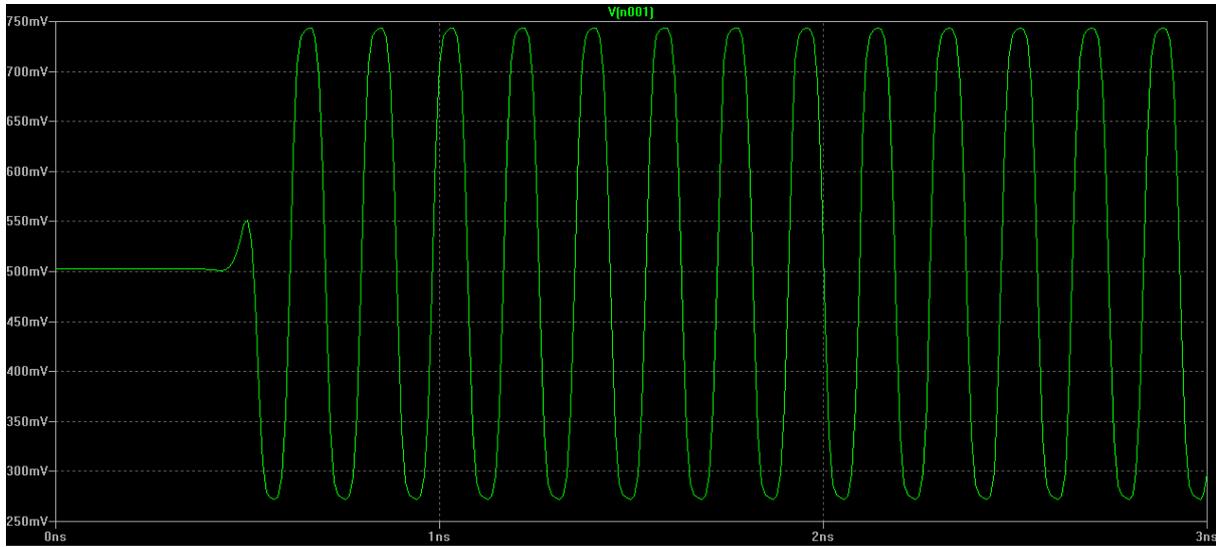
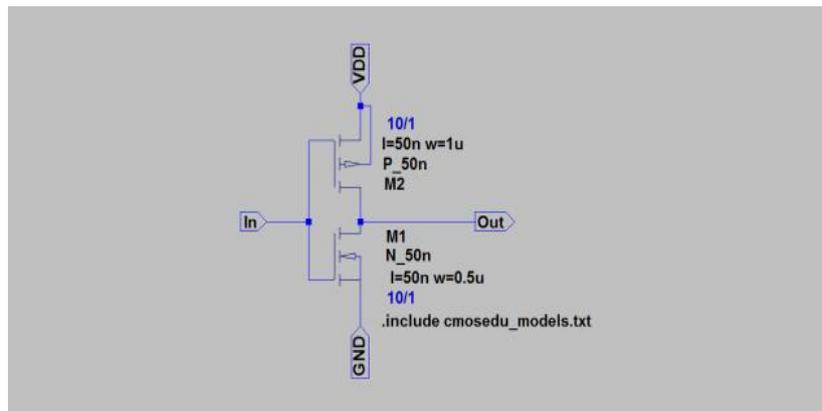**Figure 3.** Simulation results of above ring oscillator.
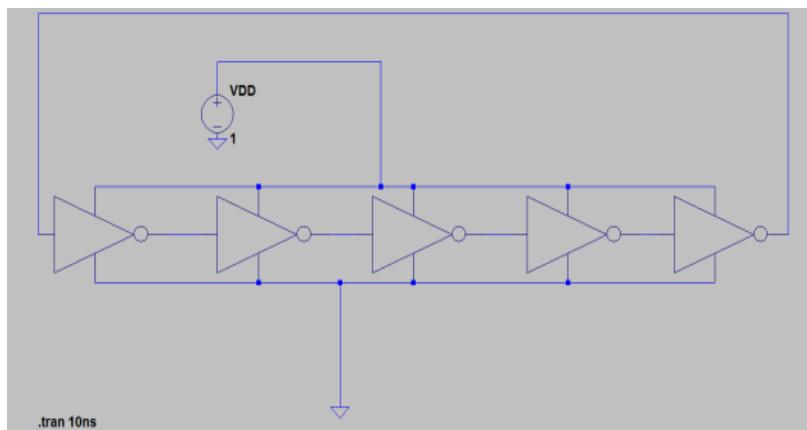

Figure 4. CMOS inverter.
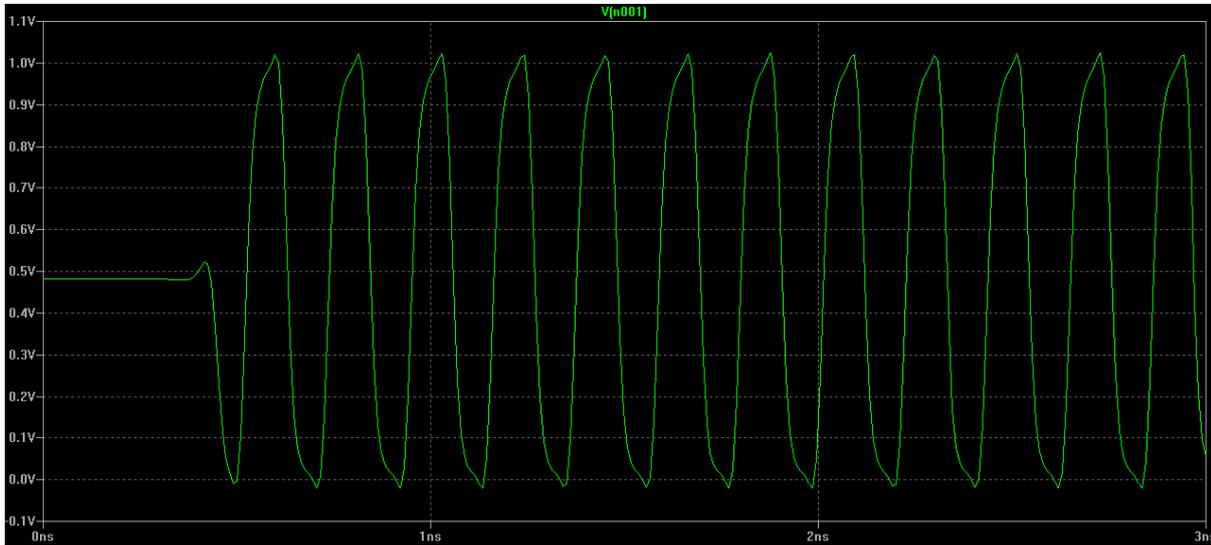

Figure 5. Ring oscillator with above inverter.

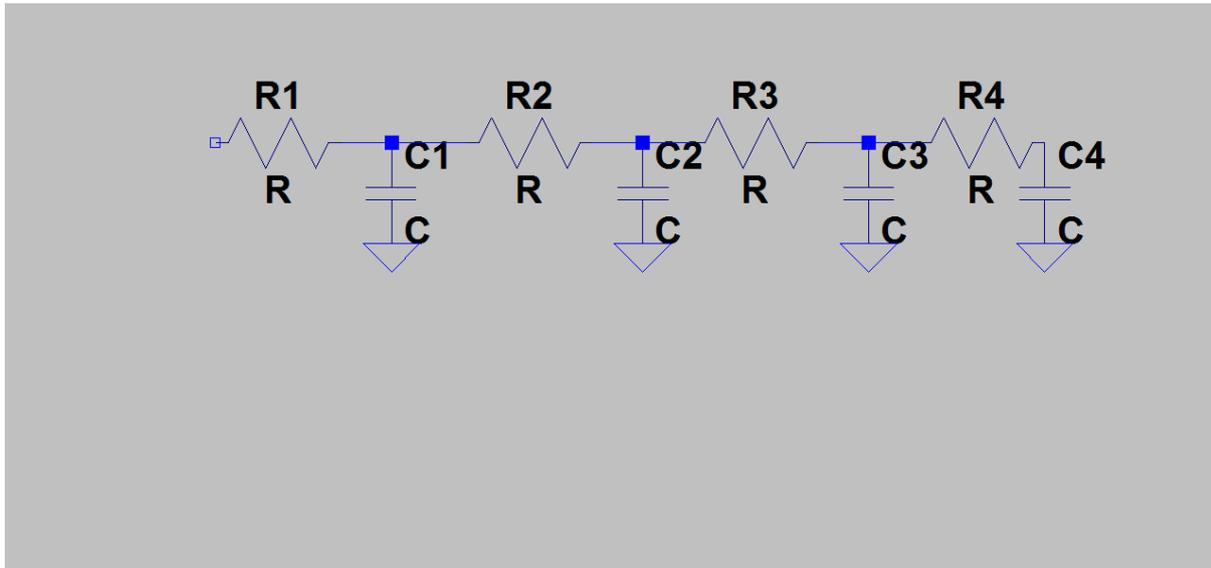**Figure 6.** Simulation results of the above ring oscillator.

Figure 7. RC ladder.

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