



# Finite Element Method Analysis of Symmetrical Coupled Microstrip Lines

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**Abstract:** In this paper, we apply the finite element method (FEM) to model and compute the capacitance and inductance per unit length matrices of symmetrical coupled microstrip lines with different dielectric constants and without. We mainly focus on modeling shielded symmetrical coupled microstrip lines and open symmetrical coupled microstrip lines, respectively. Also, we illustrate the meshing and the potential distribution of the transmission lines for the models. We compare some of our results with those obtained by other methods and found them to be in good agreement.

**Keywords:** Finite element method, Capacitance and inductance matrices, Coupled microstrip lines

## 1. INTRODUCTION

Electromagnetic propagation on multiple parallel transmission lines has been a very attractive area in computational electromagnetic in recent years. Multiple parallel transmission lines have been successfully applied and used by designers in compact packaging, semiconductor device, high speed interconnecting buses, monolithic integrated circuits, and other applications. Microstrip lines are the most commonly used in all planar circuits despite of the frequencies ranges of the applied signals. Microstrip lines are the most commonly used transmission lines at high frequencies. Quasi-static analysis of microstrip lines involves evaluating them as parallel plates transmission lines, supporting a pure "TEM" mode. Advances in microwave solid-state devices have stimulated interest in the integration of microwave circuits. Today, microstrip transmission lines have attracted great attention and interest in microwave integrated circuit applications. This creates the need for accurate modeling and simulation of microstrip transmission lines.

The computation of capacitance and inductance of microstrip transmission lines is considered essential in designing microwave and advanced integrated circuits. Accurate methods for determining the capacitance and inductance for different geometries for microstrip lines in modern design techniques have become an area of interests to scientists and researchers [1-2]. Many industrial applications depend on different interrelated

properties or natural phenomena and require multiphysics modeling and simulation as an efficient method to solve their engineering problems. Moreover, superior simulations of microwave integrated circuit applications will lead to more cost-efficiency throughout the development process. Several methods used for analyzing microstrip lines include the method of moments [3], the variational technique [4-5], the integral equation technique [6-7], the unified analytical method [8], and the Green functions method [9].

In this work, we consider two different models using FEM with different dielectric constants and without. Case A investigates the designing of shielded symmetrical coupled microstrip lines. For case B, we illustrate the modeling of open symmetrical coupled microstrip lines (two conductors between two ground planes). Also, we demonstrate the potential distribution of the transmission lines for the models and their meshing. We compare some of our results with previous investigators and find them to be close.

## 2. RESULTS AND DISCUSSIONS

The models are designed in two-dimensional (2D) using electrostatic environment. In the boundary condition of the model's design, we use ground boundary which is zero potential ( $V=0$ ) for the shield. We use port condition for the conductors to force the potential or current to one or zero depending on the setting. Also, we use continuity boundary condition between the



conductors and the dielectric layers. The quasi-static models are computed in form of electromagnetic simulations using partial differential equations.

The quasi-static analysis is valid under the assumption that,  $\frac{\partial \mathbf{D}}{\partial t} = 0$ , where  $\mathbf{D}$  is the electric displacement. Thus Maxwell's equations can be written in the following forms:

$$\nabla \times \mathbf{H} = \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{J}^e \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (4)$$

$$\nabla \cdot \mathbf{J} = 0, \quad (5)$$

where  $\mathbf{H}$  is the magnetic field,  $\sigma$  the electrical conductivity,  $\mathbf{E}$  is the electric field,  $\mathbf{v}$  is the velocity of the conductor,  $\mathbf{B}$  is the magnetic flux density,  $\mathbf{J}^e$  is an externally generated current density,  $\rho$  is the charge density,  $\mathbf{J}$  is the current density. However, the essential criterion for the quasi-static approximation to be valid is that the currents and the electromagnetic fields vary slowly. Using magnetic potential  $\mathbf{A}$  definition we get:

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (6)$$

and

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \quad (7)$$

where  $V$  is the electric potential. And by using the constitutive relation,

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \quad (8)$$

where  $\mathbf{M}$  is the magnetization, the Ampere's law now can be rewritten as

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V = \mathbf{J}^e \quad (9)$$

Where  $\mu_0$  is the permeability of vacuum. Thus, the continuity equation can be written as

$$-\nabla \cdot \left( \sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V - \mathbf{J}^e \right) = 0 \quad (10)$$

Equations (9) and (10) provide the system of the two equations for  $\mathbf{A}$  and  $V$ .

The dimension of the coefficient capacitance matrix is proportional to the sum of widths of every dielectric layer and the parameters of all conductors. This results in long

computing time and large memory especially when the structure to be analyzed has many layers and conductors [10]. The short-circuit capacitances  $C_{ij}^s$ , defined as in [11]:

$$Q_i = C_{i1}^s V_1 + \dots + C_{ii}^s V_i + \dots + C_{iN}^s V_N \quad (11)$$

and the two-terminal capacitances  $C_{ij}$  is defined as

$$Q_{ii} = C_{ii} V_i \quad (12)$$

$$Q_{ij} = C_{ij} (V_i - V_j), \quad i \neq j \quad (13)$$

where  $Q_i$  is the charge per unit length on a conductor  $i$ ,  $V_j$  is the potential of a conductor  $j$ , with respect to the ground (conductor  $N+1$ ),  $Q_{ii}$  is the partial charge on conductor  $i$  due to a voltage difference between conductor  $i$  and the reference (ground) conductor,  $Q_{ij}$  is the partial charge on conductor  $i$  due to a voltage difference between conductors  $i$  and  $j$ .

To conclude the relationship between the short-circuit capacitances and the two-terminal capacitances as:

$$C_{ij} = -C_{ij}^s \rightarrow C_{ii} = \sum_{j=1}^N C_{ij}^s \quad (14)$$

Furthermore, we use one port at a time as the input to evaluate the matrix entries. With the Forced Voltage method, the capacitance matrix entries are computed from the charges that result on each conductor when an electric potential is applied to one of them and all the others are set to ground. The matrix is defined as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} = \begin{bmatrix} C_{11}V_1 + C_{12}V_2 + \dots + C_{1N}V_N \\ C_{21}V_1 + C_{22}V_2 + \dots + C_{2N}V_N \\ \vdots \\ C_{N1}V_1 + C_{N2}V_2 + \dots + C_{NN}V_N \end{bmatrix} \quad (15)$$

For example, using port 2 as the input will provide the entries of the second column:  $C_{12}, C_{22}, \dots, C_{N2}$ .

Now, the inductance and capacitance of coupled transmission lines are related as:

$$[L] = \mu_0 \varepsilon_0 [C_o]^{-1}, \quad (16)$$

where  $[L]$  = inductance matrix,  $[C_o]^{-1}$  = the inverse matrix of the capacitance of the multiconductor transmission line when all dielectric constants are equal to 1,  $\varepsilon_0$  = permittivity of free space or vacuum,  $\mu_0$  = permeability of free space or vacuum.

The models designed with finite elements are unbounded (or open), meaning that the electromagnetic fields should extend towards infinity. This is not possible because it would require a very large mesh. The easiest approach is just to extend the simulation domain “far enough” that the influence of the terminating boundary conditions at the far end becomes negligible. In any electromagnetic field analysis, the placement of far-field boundary is an important concern, especially when dealing with the finite element analysis of structures which are open. It is necessary to take into account the natural boundary of a line at infinity and the presence of remote objects and their potential influence on the field shape [12]. In our simulations for the open case, the open two conductors structure is surrounded by a  $W \times H$  shield, where  $W$  is the width and  $H$  is the thickness.

A. Shielded Symmetrical Coupled Microstrip Line

Figure 1 shows the cross section for shielded two-strip conductors in a homogenous medium with its parameters:

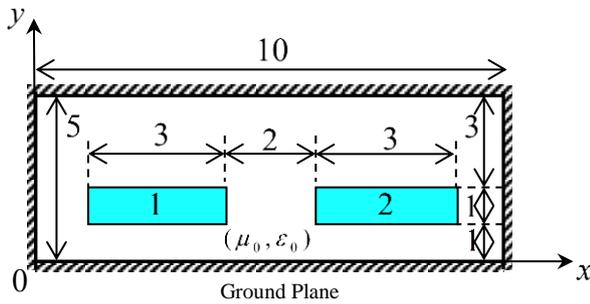


Figure 1. Cross-section of shielded two-strip conductors in a homogenous medium

From the model, we generate the finite elements mesh as in Fig. 2. Table I shows the statically properties of the mesh. Figure 3 shows the two-dimensional (2D) surface potential distribution with port 2 as input. While, the contour of electric potential (V) and streamline of electric field plots of the model are presented in the Figures 4 and 5 respectively.

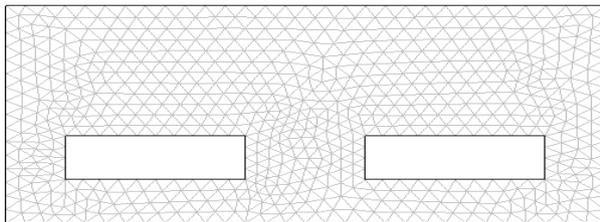


Figure 2. Mesh of shielded two-strip conductors in a homogenous medium

Table I. Mesh statistics of shielded two-strip conductors in a homogenous medium

Items	Value
Number of degrees of freedom	3019
Total number of mesh points	717
Total number of elements	1288
Triangular elements	1288
Quadrilateral elements	0
Boundary elements	148
Vertex elements	12
Minimum element quality	0.8340
Element area ratio	0.3185

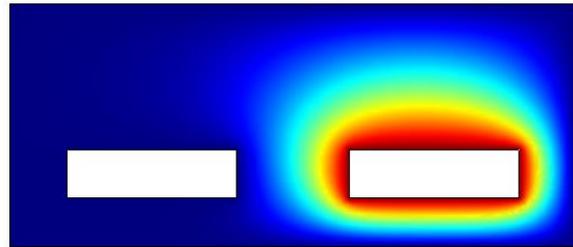


Figure 3. 2D surface potential distribution of shielded two-strip conductors in a homogenous medium

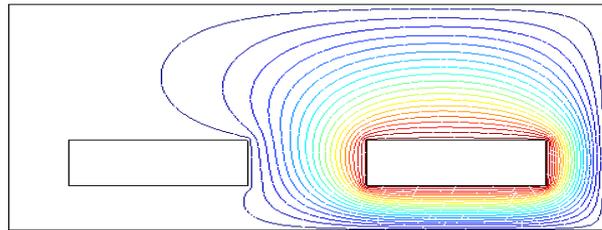


Figure 4. Contour plot of shielded two-strip conductors in a homogenous medium



Figure 5. Streamline plot of shielded two-strip conductors in a homogenous medium

Table II shows the finite element results for the self and mutual capacitance per unit length of shielded symmetrical coupled microstrip lines without dielectric constant. The results in Table II are compared with the

work of previous investigations using Green function method [9]. They are in good agreement.

TABLE II. VALUES OF THE CAPACITANCE COEFFICIENTS (IN PF/M) FOR SHIELDED SYMMETRICAL COUPLED MICROSTRIP LINE

Capacitance	Green Function Method	FEM
$C_{11}$	70.06	70.63
$C_{12}$	-5.776	-5.901
$C_{21}$	-5.776	-5.901
$C_{22}$	70.06	70.63

We extend the analysis to compute the self- and mutual inductance per unit length due to its importance role in the high-speed digital circuits or simultaneous switching noise generates disparities among local found potentials in different packages [13] using equation (16):

$$[L] = \begin{bmatrix} 158.4 & 13.2 \\ 13.2 & 158.4 \end{bmatrix} nH / m \quad (17)$$

Table III shows the finite element results for the self and mutual capacitance per unit length of shielded symmetrical coupled microstrip lines with different dielectric constants, 3.2, 4.3, and 6.8. The results in Table III are show that when we increase the dielectric constant value the self capacitance per unit length are increases, while the values of the mutual capacitance per unit length are decreases.

TABLE III. VALUES OF THE CAPACITANCE COEFFICIENTS (IN PF/M) FOR SHIELDED SYMMETRICAL COUPLED MICROSTRIP LINE IN DIFFERENT DIELECTRIC CONSTANSTS

Capacitance	$\epsilon_r = 3.2$	$\epsilon_r = 4.3$	$\epsilon_r = 6.8$
$C_{11}$	226.06	303.77	480.38
$C_{12}$	-18.88	-25.37	-40.13
$C_{21}$	--18.88	-25.37	-40.13
$C_{22}$	226.06	303.77	480.38

We use this model from [9] to compare the capacitance per unit matrix of the model results using FEM with the Green functions method. Here we computed the inductance per unit matrix of the model, which was not done by the other methods. Also, we studied the model with different dielectric constants and we identify for the model the mesh, two-dimensional (2D) surface potential distribution, contour plot, streamline plot, and potential distribution of the model along the line from

$(x, y) = (0, 0)$  to  $(x, y) = (10, 5)$ , which were not done by the other methods.

Fig. 6 shows the comparison analysis of potential distribution of the model with and without dielectric substrate. It observed that the peak value of electric potential is approximately same as the different dielectric is placed in the substrate.

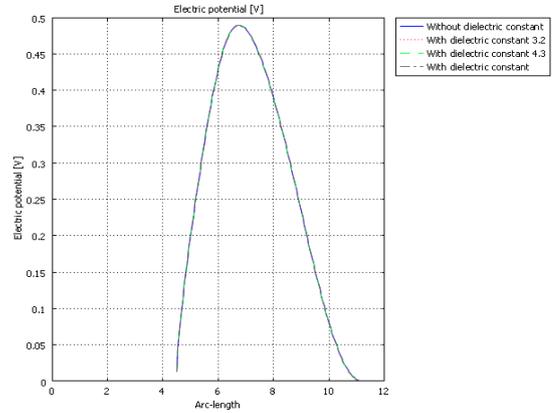


Figure 6. Potential distribution of the model with and without dielectric substrate

### B. Open Symmetrical Coupled Microstrip Line

Figure 7 shows the cross section of the open symmetrical coupled microstrip line with its parameters.

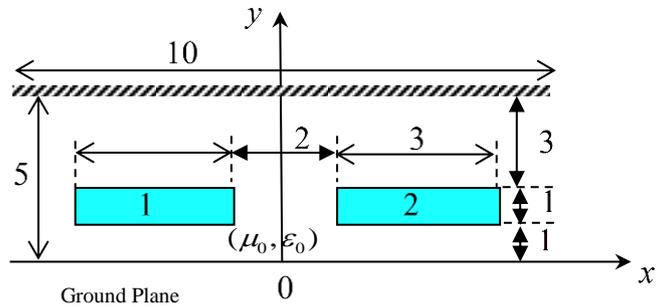


Figure 7. Cross-Section of symmetrical coupled microstrip line in a homogeneous medium between ground planes

In our simulations for this case, we surround the structure by a  $20 \times 5$  shield. From the model, we generate the finite elements mesh as in Fig. 8. Table IV shows the statically properties of the mesh. Figure 9 shows the two-dimensional (2D) surface potential distribution with port 2 as input. While, the contour of electric potential (V) and streamline of electric field plots of the model are presented in the Figures 10 and 11 respectively.

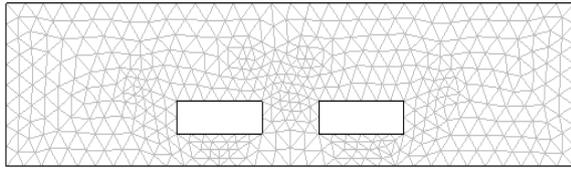


Figure 8. Mesh of symmetrical coupled microstrip line in a homogeneous medium between ground planes

TABLE IV. MESH STATISTICS OF SYMMETRICAL COUPLED MICROSTRIP LINE IN A HOMOGENEOUS MEDIUM BETWEEN GROUNDS PLANES

Items	Value
Number of degrees of freedom	2001
Total number of mesh points	472
Total number of elements	836
Triangular elements	836
Quadrilateral elements	0
Boundary elements	110
Vertex elements	12
Minimum element quality	0.7183
Element area ratio	0.1714

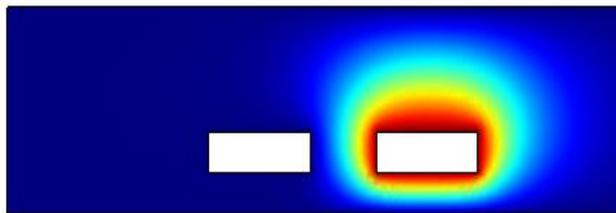


Figure 9. 2D surface potential distribution of symmetrical coupled microstrip line in a homogeneous medium between ground planes

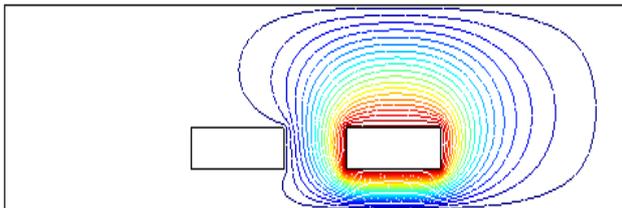


Figure 10. Contour plot of symmetrical coupled microstrip line in a homogeneous medium between ground planes

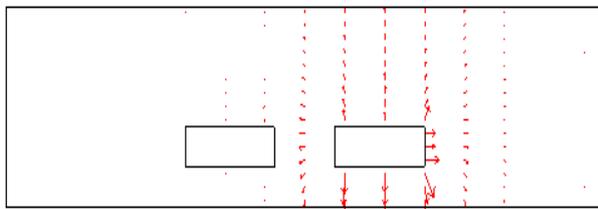


Figure 11. Streamline plot of symmetrical coupled microstrip line in a homogeneous medium between ground planes

Table V shows the finite element results for the self and mutual capacitance per unit length of the open symmetrical coupled microstrip lines. The results in Table V are compared with the work of previous investigations. They are in good agreement.

TABLE V. VALUES OF THE CAPACITANCE COEFFICIENTS (IN PF/M) FOR OPEN SYMMETRICAL COUPLED MICROSTRIP LINE

Capacitance	Ref. [3]	Ref. [7]	Ref. [9]	FEM
$C_{11}$	62.33	63.07	63.10	63.766
$C_{12}$	-5.931	-5.866	-5.851	-5.986
$C_{21}$	-5.931	-5.866	-5.851	-5.986
$C_{22}$	62.33	63.07	63.10	63.766

In addition, we extend the analysis to compute the inductance per unit matrix using equation (16):

$$[L] = \begin{bmatrix} 175.8 & 16.5 \\ 16.5 & 175.8 \end{bmatrix} nH/m \quad (18)$$

Table VI shows the finite element results for the self and mutual capacitance per unit length of open symmetrical coupled microstrip lines with different dielectric constants, 3.2, 4.3, and 6.8. The results in Table VI are show that when we increase the dielectric constant value the self capacitance per unit length are increases, while the values of the mutual capacitance per unit length are decreases.

TABLE VI. VALUES OF THE CAPACITANCE COEFFICIENTS (IN PF/M) FOR OPEN SYMMETRICAL COUPLED MICROSTRIP LINE IN DIFFERENT DILECTRIC CONSTANSTS

Capacitance	$\epsilon_r = 3.2$	$\epsilon_r = 4.3$	$\epsilon_r = 6.8$
$C_{11}$	204.07	274.21	433.63
$C_{12}$	-19.16	-25.74	-40.71
$C_{21}$	-19.16	-25.74	-40.71
$C_{22}$	204.07	274.21	433.63

This model was presented in [14] to compare the capacitance per unit matrix of the model results using FEM with the method of moments and the variational technique. Here we use different parameters to compare our capacitance per unit matrix of the model results using the method of moment, the integral equation technique, and the Green functions method. Also, we computed the inductance per unit matrix of the model, which was not done by the other methods and in [14]. In addition, we

studied the model with different dielectric constants and we identify for the model the mesh, we identify for the model the contour plot, streamline plot, two-dimensional (2D) surface potential distribution, and potential distribution of the model along the line from  $(x, y) = (0, 0)$  to  $(x, y) = (20, 5)$  which was not done by the other methods. In [15] one of the models was four-line symmetric microstrip with a two-layer substrate which is different from this model.

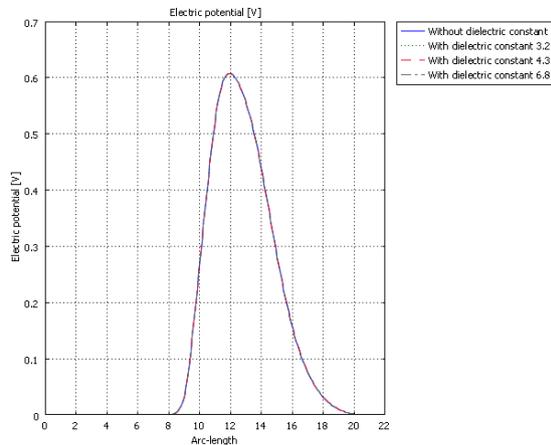


Figure 12. Potential distribution of the model with and without dielectric substrate

### 3. CONCLUSION

We have successfully presented finite element analysis and modeling of shielded and open symmetrical coupled microstrip lines with different dielectric substrate and without. In addition, we computed the capacitance and inductance matrices and identified the potential distribution of the models. The results obtained efficiently using FEM for the capacitance agrees well with those found in the other methods.

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